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Reconciling Diverging Views on Mineral Depletion: A Modified Cumulative Availability Curve Applied to Copper Resources*

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ABSTRACT

Mineral depletion is a perennial concern in natural resources management. Despite a multiplicity of perspectives, most analyses fall largely into one of two groups. Studies in the first group take the fixed quantity of any material in the earth as the starting point for analysis, focus primarily on physical stocks and flows of mineral resources, and assess when society will run out of a resource or when production and use will peak (the fixed-stock viewpoint). Studies in the second group start by noting the heterogeneous nature of mineral resources and our incomplete knowledge of the quantity and quality of minerals in the earth, focus on the dynamic nature of mineral availability, and assess the ability of society to adjust to mineral depletion at existing mines though mineral exploration and mine development, technological innovation, and substitution (the opportunity-cost viewpoint). This paper seeks to reconcile these diverging perspectives by developing a modified cumulative availability curve (CAC) - which combines physical stocks and flows (from the fixed-stock perspective) with geologic stock uncertainty, different demand scenarios and extraction costs (from the opportunity-cost perspective). When applied to copper resources, the modified CAC suggests that copper demand is likely to be satisfied until about 2075 from known deposits. Thereafter, the ability of mineral explorers to discover previously undiscovered deposits, as well as efforts at substitution and improving recycling and re-use of copper, will importantly influence whether copper demand is satisfied.

JEL **classifications: Q3**
Keywords: peak minerals, mineral depletion, cumulative availability curve, copper.

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1. Introduction

Minerals have been an intrinsic part of human development, from the stone age to what is being called the silicon age of the 21st century. Minerals stand as major inputs in technological development and in the improvement of human living standards. Given the broad use of minerals in modern society, long-term mineral depletion is a critical issue for policy makers and society.

Concern about depletion of mineral resources is not new. It has been a recurring societal concern. In the middle 1800s, for example, W.S. Jevons explored the exhaustion of coal in Britain (Jevons 1865). In the late 1800s and early 1900s, dramatic increases in the use of fertilizers in commercial agriculture led to fears about the adequacy of naturally occurring sources of nitrogen, which at the time came from animal dung, urine and Chilean nitrate deposits (Hager 2008, Smil 2004). In the years following World War II, concerns focused on a wide range of mineral raw materials and energy, given the significant use of mineral and energy resources during the war (The U.S. President's Materials Policy Commission 1952). In the 1970s, attention returned to energy and minerals, in this case concerns about reliability of foreign sources of oil and some nonfuel minerals, as well as long-term adequacy of nonrenewable resources generally and implications for economic growth (V. Smith 1979). In each case, society adjusted to perceived scarcities through one or a combination of three approaches: developing new sources of mineral and energy resources, using resources more efficiently and developing substitute materials or sources of energy.

A number of recent studies focus on the relationships between and among mining, sustainable development and long-term resource depletion. Dubiński (2013) argues that rational acquisition and use of natural resources are central to sustainable mineral development.. Ali et al. (2017) take advantage of peak models of resource extraction to suggest that mineral depletion – as reflected by declining production and use of copper - may start to occur sooner than expected, urging policy makers to take coordinated and planned actions at a global scale, intervening in mineral markets to promote sustainable development goals. In response, Tilton et al. (2018) criticize both the use of peak models to assess long-term mineral availability and depletion and the call for strong government intervention to mitigate future shortages beyond interventions to control environmental damages and to correct the tendency of private actors to underinvest in technological innovation from society's perspective.

Broadly speaking, approaches to assessing the threat of mineral depletion fall into two categories: "fixed stock" and "opportunity cost" approaches, where consensus is still hard to reach in even basic definitions (Segura-Salazar and Tavares 2018). The fixed-stock approach is dominantly a physical view of natural resource scarcity, while the opportunity-cost approach is largely economic (Barbier 2019). Reflecting the physical approach, Gordon et al. (2006) argue that physical stocks of metals will be unable to sustain the modern quality of life, suggesting an absolute scarcity of minerals. Moreover, prices will not result in real warnings since price trends are not different between abundant and scarce minerals and markets do not reflect external mining costs (Henckens, et al. 2016). In this context, peak models arise as an increasingly preferred approach for assessing mineral depletion using different physical stock measures (Ali, et al. 2017, Calvo, Valero and Valero 2017, Northey, et al. 2014, Prior, et al. 2012). Reflecting the opportunity-cost approach, many mineral economists argue that peak models for mineral resources are inherently flawed and, as an alternative, suggest a cumulative availability curve (CAC) to assess long-term availability (Tilton and Lagos 2007, Tilton and Guzman 2016, ch. 9, Tilton, et al. 2018, J. Tilton 2018).

The fixed-stock and opportunity-cost approaches differ significantly in the inferences they draw on how to respond to the threat of mineral scarcity. While the former urges society to change the pace of development, the latter claims that prices and costs will provide incentives to offset depletion. The diverging positions appear in virtual stalemate: economists would disregard physical models that do not include how markets respond to scarcity, and natural scientists do not feel comfortable waiting for markets to provide signals to inform decisions by governments and private actors.

The purpose of this paper is to review the two approaches for assessing the threat of mineral depletion and develop a reconciled methodological approach. Toward this end, we build a theoretical model for long-term supply and demand for minerals. The formal model allows us to be explicit about assumptions behind peak models and CAC applications. Then, we suggest modifying the CAC in two main aspects to help reconcile these approaches. First, we propose to emphasize separate demand scenarios for more-developed and less-developed regions. Second, we include different supply units depending on economic and geologic stock uncertainty, based on reserves, resources and undiscovered deposits. We finally apply the modified CAC to copper resources and compare our results with previous peak models.

The CAC has fewer empirical applications compared to peak models (Calvo, Valero and Valero 2017, J. Tilton 2018). Since CAC's theoretical development in 1987 only a few authors have studied long-term depletion for specific resources like lithium (Yaksic and Tilton 2009), tellurium (Redlinger, et al. 2013), conventional and unconventional oil (Aguilera, et al. 2009, R. F. Aguilera 2014), thorium (Jordan, et al. 2015), indium (Lokanc, Eggert and Redlinger 2015), gallium (Frenzel, et al. 2016) and iron ore (Jasiński, Meredith and Kirwan 2018). In practice, there is not a clear point in the CAC to mark depletion, contrasting with the "peak" in peak models. Despite previous applications, no author has clearly formalized the general methodological assumptions of the CAC, nor dedicated significant effort to demand's role. Additionally, most CAC applications have been focused on minor minerals, which may hinder its application to more economically important minerals like copper.

Section two reviews the major depletion studies using peak models and CACs. We find that extraction costs driving long-term mineral depletion is common in both approaches. Both rely on detailed information on reserves and resources to build their models. Demand in both cases is assumed as perfectly price inelastic (that is, consumption is not affected by price changes) and increasing. Section three proposes methodological changes for supply and demand in the CAC to reconcile differences with peak models. Finally, the last section applies the modified CAC to copper resources, suggesting that known reserves and resources are responsive enough to meet demand until 2075. This result highly contrasts with peak copper models, which signal peak copper before 2050.

2. Two models for mineral depletion

Peak models and the CAC differ fundamentally in the unit of analysis. The former focus on physical units to estimate availability such as tonnage, kilograms or grades. On the other hand, the latter is primarily concerned with costs and prices, economic resources that society needs to give up for an additional unit of mineral resources (Tilton and Guzman 2016, ch. 9). In practice, there are two main similarities and two differences between the two approaches. The approaches are similar in that supply drives long-term depletion and lack of complete geologic information is a major limitation. The approaches are different in that peak models use peak production as a clear signal of impending depletion, while CAC does not have a clear counterpart as an indicator of depletion. Moreover, CAC models acknowledge that markets will adapt as depletion drive prices up, either encouraging exploration or by demand substitution. This adaptive process is not part of peak models, a common critique from the opportunity-cost advocates.

2.1 Peak models

The physical view usually turns to Hubbert's theory of peak production to assess mineral depletion. Peak models are based on a bell-shaped profile in the extraction of a fixed stock (Hubbert 1956). The simplicity of the theory is the main driver behind its widespread application, despite critiques from opportunity-cost supporters. There is only one input that defines peak production: the Ultimate Recoverable Resource (URR) defines the entire supply over time in peak models. The URR is an estimate of the total mineral resources that society will recover from mineral deposits, both historically and into the future (Giurco, et al. 2012). In its simplest form (Calvo, Valero and Valero 2017), mined supply at every period t (S_t) is given by equations (1) and (2). In these equations, b and t_{peak} (the peak year) are the regression parameters to be adjusted to fit historical data, and the URR is assumed by construction.

$$S_t = \frac{URR}{b} e^{-\frac{1}{2}\left(\frac{t-t_{peak}}{b}\right)^2} \tag{1}$$

$$S_{t_{peak}} = \frac{URR}{b\sqrt{2\pi}} \tag{2}$$

Peak models implicitly assume that other determinants of supply (price, technology, exploration or input costs) are irrelevant to assess long-term depletion (J. Tilton 2018). In practice, peak models allow different URR scenarios, but changing the URR does not generate much change in the peak year t_{peak} (Northey, et al. 2014, Sverdrup, Ragnarsdottir and Koca 2014). Many authors have tried to incorporate more variables to the simplest Hubbert model. Sverdrup et al. (2014) develop a complex world dynamic model combining population, recycling, markets and mining supply. In their model, increasing demand after the peak should be met by increasing recycling rates. Giurco et al. (2012) propose a detailed assessment at a mine level, where production decisions follow a trapezoid-shaped production profile over time. Northey et al. (2014) build on the Giurco et al. (2012) research, including ore grade as the main driver defining what deposits will be mined first. Quantity demanded in peak models is not a relevant variable as long as it is greater than or equal to production from the peak function. The demand condition is guaranteed by having a non-decreasing per capita demand. In the long-term, increasing recycling and exploration is expected to fill the gap between increasing demand and constrained peak supply (Northey, et al. 2014, Sverdrup, Ragnarsdottir and Koca 2014).

Modifying the simplest peak model does not result in major changes in results, generating a sense of theory robustness. For example, in the case of peak copper models, previous literature agrees on a copper shortage over the next 20 to 30 years, slightly dependent on the URR value (Bardi and Pagani 2007, Giurco, et al. 2012, Laherrére 2010, Northey, et al. 2014, Sverdrup, Ragnarsdottir and Koca 2014). Therefore, there is little to gain by adding more dynamic components to the simplest peak model (Calvo, Valero and Valero 2017).

Critiques of peak models to assess minerals long-term depletion have been widely discussed by supporters of the opportunity-cost paradigm. First, peak models do not consider the effect of technology increasing availability of reserves, resources and undiscovered deposits

(Kharitonova, Mikhailo and Matsko 2013). Second, peak models usually ignore market behavior such as, for example, that peak production for minerals may occur for political, health or environmental restrictions, rather than physical depletion, and that materials intensity of use declines as countries develop (Criqui 2013, Crowson 2011, Ericsson and Söderholm 2013, Tilton and Guzman 2016, ch. 9). Third, geological stock uncertainty created by lack of discovery data and unknown deposits does not affect the dynamic behavior of peak models. Additionally, peak models erroneously consider reserves and resources as fixed stock measures (May, et al. 2012, Meinert, Robinson and Nassar 2016, Wellmera and Scholz 2018). As a result, peak model assumptions are generally strong, questionable and biased towards a more pessimistic prediction of mineral depletion (J. Tilton 2018). Nevertheless, peak models have an impact on the scientific community concerned about sustainable development in the mineral industry (Ali, et al. 2017, Calvo, Valero and Valero 2017) or when it comes to think strategically about resources (Chapman 2014).

2.2 The Cumulative Availability Curve

The opportunity-cost approach focuses on what society needs to give up for an additional unit of a mineral resource. The cumulative availability curve (CAC) is an alternative to assess mineral depletion along with trends in real prices, trend in real extraction costs and the value of marginal reserves. The CAC represents all known geological sources for a mineral and their full extraction costs over all time. In the CAC, demand determines how quickly we extract available mineral resources (Tilton and Guzman 2016, ch. 9). Mineral depletion will drive prices up, discouraging consumption, increasing substitution and recycling, and encouraging new supply sources by exploration or technology. It is worth noting that the CAC uses all available information given time, while technology, input costs, discoveries and other supply determinants remain fixed (J. Tilton 2002). The CAC implies that known lower cost deposits will be mined first in the long-term. The concept is common both in peak models and the CAC, but extraction cost information is not available for uncertain sources, like resources or undiscovered deposits. To solve the data problem, peak models assume that extraction costs rise because of a decline in a deposit ore grade (Henckens, et al. 2016, Northey, et al. 2014, Prior, et al. 2012, Vieira, et al. 2012). Nonetheless, a declining ore grade is a poor depletion predictor because of the endogenous relationship between ore grade and economic conditions (Crowson 2012, Tilton and Guzman 2016, ch. 7, West 2011, Wood Mackenzie 2015).

Formally, equation (3) indicates that the CAC is a stock variable, adding up all known deposits or mineral sources i (R_i) at a time t, as a function of the price to incentivize their extraction (p), keeping all other supply determinants fixed. Equation (4) reflects demand, that it is assumed as relatively inelastic at every period t but changing in response to income and population growth (g_t) and material substitution efforts (s_t). Is it expected that $\frac{\partial f}{\partial s} < 0$, because more substitution effort should decrease metal consumption. Additionally, demand can also change over time as an effect of consumer preferences or new technology.

$$CAC_t(p) = \sum_i R_{i,t}(p) \tag{3}$$

$$D_t = f(g_t, s_t, t) \tag{4}$$

The CAC indicates that future price is determined by future extraction costs, and demand will define how quickly we deplete non-renewable stocks (Tilton and Guzman 2016, ch. 9). The CAC approach assesses depletion in three steps. First, expected future demand is summed from the current period (t_0) up to a time in the future (t_f). Second, the price required to supply that amount from current reserves is obtained using the inverse of the CAC. Finally, if the resulting price is in the steeper section of the CAC following equation (5) and price needs to greatly increase to stimulate further supply, then there is a major threat from depletion (J. Tilton 2018). However, moving into a steeper section of the CAC should also trigger discoveries and substitution efforts (Tilton and Guzman 2016, ch. 9).

$$\frac{\partial CAC}{\partial p}\left(CAC^{-1}\left(\sum_{t_0}^{t_f} D_t\right)\right) \tag{5}$$

The opportunity-cost paradigm does not generate complete consensus on how to assess mineral depletion. First, critics are less confident in the role of markets providing necessary incentives to offset depletion (J. Tilton 2002) – importantly due to the arguably high external social and environmental costs of mining that are not fully internalized by markets (Segura-Salazar and Tavares 2018). Second, assuming that geologically scarce minerals are more likely to face depletion than geologically abundant minerals, then price trends should be different between them. However, price trends are not different between geologically abundant and scarce minerals., failing to signal mineral depletion (Henckens, et al. 2016). The implication is that responding to mineral depletion requires international coordination to assure supplies of geologically scarce minerals for future generations (Henckens, Driessen and Ryngaert, et al. 2016, Henckens, Ryngaert, et al. 2018). Third, the opportunity-cost paradigm does not seem concerned about exponential growing demand and overstates the role of technology offsetting depletion (Gordon, Bertram and Graedel 2006, Humphreys 2013).

Despite critiques, the CAC is useful to assess potential depletion shortages (Yaksic and Tilton 2009). **Figure 1** illustrates the applicability of the CAC. For example, **Figure 1** a) indicates that minerals without discontinuities in the CAC and gradual rising costs are unlikely to suffer from rapid scarcity as demand expands; rather society is likely to see scarcity emerging and allowing time for response through, for example, technological innovation, new supplies, or substitution). Nonetheless, discontinuous jumps or sharp increases in the CAC, as depicted in **Figure 1** b), c), and d) suggest that scarcity may emerge suddenly, catching society off guard with insufficient time to respond appropriately. **Figure 1** d) shows a rising slope, becoming inelastic as we approach to stock depletion (R. F. Aguilera 2014).

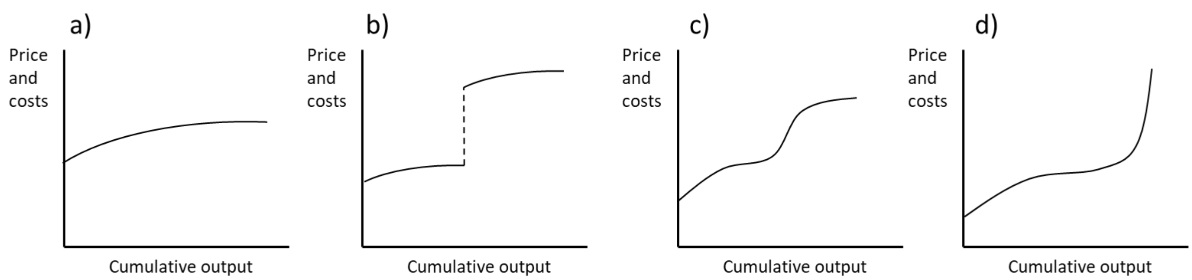

Figure 1 Illustrative cases of the CAC. (a) Slowly rising slope due to gradual increase in costs. (b) Discontinuity in slope due to jump in costs. (c) Sharply rising slope due to rapid increase in costs. (d) Rising slope rapidly becomes inelastic at maximum cumulative output constraint. Source: Tilton and Skinner (1987) and Aguilera (2014).

The number of actual CAC estimates is limited, likely due to data limitations (Tilton and Guzman 2016, ch. 9). Additionally, the CAC is not expected to provide insights about how quickly society consumes available stocks to point expected depletion. However, the CAC is a more general and flexible approach than peak models, allowing the consideration of different factors affecting mineral depletion (J. Tilton 2018).

3. Building consensus in depletion models

The analysis of peak models and CAC applications support two major common concepts: supply and extraction costs drive depletion in the long-term, and there is uncertain geological information. In peak models, rising and then declining production (over the longer term) is the result of gradual increases in the in the production cost of the resource (Bardi and Pagani 2007). Ore grade decline is also used as a proxy to reflect increasing production costs and environmental impacts (Northey, et al. 2014, Sverdrup, Ragnarsdottir and Koca 2014). Therefore, even if market prices are not a satisfying measure of scarcity, both sides agree that extraction costs from various sources at current technology may provide useful insights about future mineral availability (Henckens, et al. 2016, J. Tilton 2018). Geological information relies on data of reserves, resources and undiscovered deposits, which are stocks concepts; these data often are misunderstood to mean all the mineral resource there is, which is incorrect (Meinert, Robinson and Nassar 2016). A reconciled view of mineral depletion should consider geological stock uncertainty based on the resource classification and their different expected extraction costs. Given common concepts the CAC (as a more general approach) can be modified to provide proper information about future mineral availability and their expected extraction costs (Henckens, et al. 2016, Tilton and Guzman 2016, ch. 9).

Besides common concepts, consensus requires building on critiques arising from each paradigm. First, the opportunity-cost paradigm highlights the uncertain consumption forecast, changing demand with the level of development and the demand response to offset mineral depletion (Crowson 2011, Tilton and Guzman 2016, ch. 2). Second, fixed-stock supporters criticize the opportunity-cost paradigm's inability to pinpoint mineral depletion, heavily relying on technological change and market forces. Then, it is required to find a clearer way to mark depletion in the CAC.

In the following sections we provide a framework to modify the CAC in two main aspects: demand and supply. First, the demand side should consider a changing per capita demand as countries develop in the long-term. Second, on the supply side, geological stock uncertainty requires modifying the CAC to including a certainty level on extractable or recoverable resources, based on the geological classification for mineral resources. Both modifications are necessary to construct a clear point to mark depletion in the CAC.

3.1. Demand

Long-term metal consumption in economic and peak models relies to a significant extent on variants of the intensity-of-use hypothesis (IU), either analyzing metal consumption by unit of income (GDP) or per capita. The original IU hypothesis suggests that metal consumption changes with economic development and per capita income (Tilton and Guzman 2016, ch. 2). In practice, the ration of metal consumption to total income will follow an inverted U shape, which is supported by historical evidence (Wårell and Olsson 2009). In other study, Sverdrup et al. (2014) acknowledge that copper per capita consumption depends on per capita GDP, but they consider a stable demand at approximately 10 kg/person when income per capita exceeds $25,000. However, they do not consider that IU for individual countries may decline on average as per capita GDP increases (Cuddington and Zellou 2012). Similarly, Müller, Wang and Duval (2011) find a saturation consumption level between 8 to 12 tons of iron per capita when studying future availability of iron.

In the case of copper, **Figure 2** shows historical data of metal per capita consumption following an inverted U path in more developed regions[2]. The trend suggests that global per capita metal consumption would not increase unbounded over the next century and it is expected to converge at the average level of developed countries. The previous assumption

[2] For consistency, we use the UN convention for more developed regions, including Europe, Northern America, Australia/New Zealand and Japan (United Nations 2017).

requires maintaining current material composition of good and services, consumer preferences and technological levels.

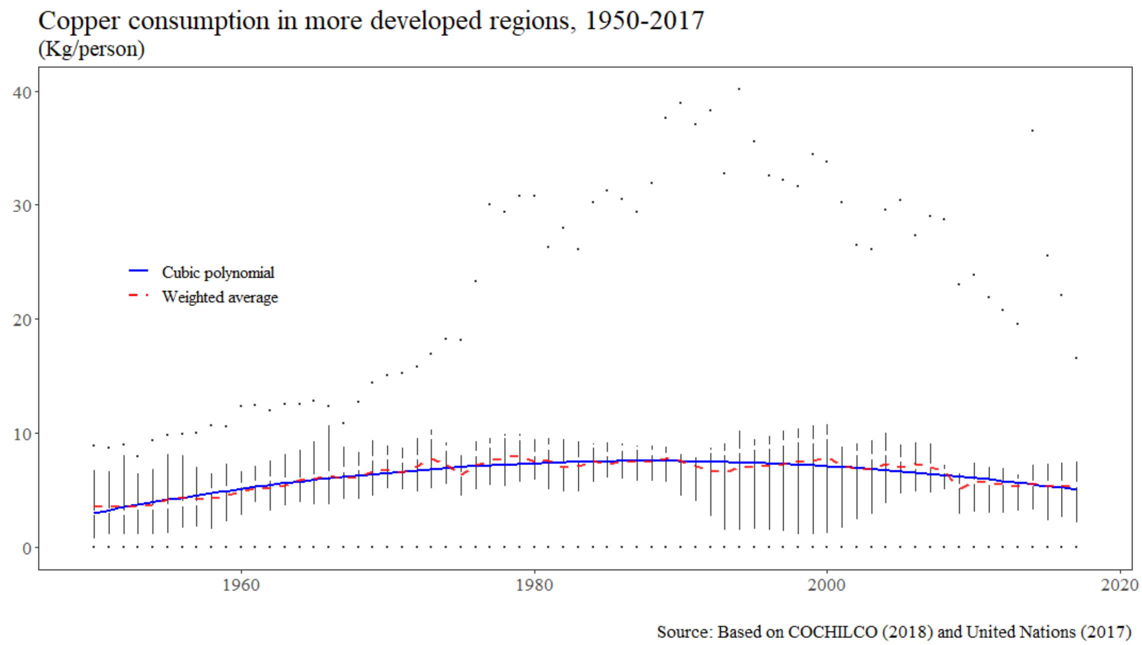

Figure 2 Per capita copper consumption for countries in more developed regions, 1950-2017. Points mark maximum and minimum per capita copper consumption every year and lines represent the interquartile range. Weighted average takes total consumption of more developed regions every year divided by their aggregated population.

Using a variant of IU hypothesis, defining per capita metal consumption, provides an approach to develop a common view on mineral depletion, given its simplicity and use in previous applications. The methodology can be refined to provide a more robust analysis in two ways. First, differentiate consumption from more and less developed countries in the analysis, defining various demand scenarios based on different saturation points. Separating more and less developed countries allows to consider consumption patterns as countries develop. Besides, demand scenarios provide a range of variability for unexpected future outcomes. Second, demand should respond to mineral scarcity as prices rise. Future market behavior may respond in ways impossible to know but we can infer when substitution efforts are likely to trigger reductions in demand. Even if metal demand is inelastic in the short-term with respect to price changes, we should observe a greater response in the long-term.

Following previous works using variants of the IU hypothesis (Cuddington and Zellou 2012, Sverdrup, Ragnarsdottir and Koca 2014), the convergent hypothesis implies that, on average, less developed regions change their consumption level to approach the consumption level of more developed region. We propose to estimate the growth of the per capita consumption (CC) from less developed regions as proportional to current consumption and the gap with more developed regions, as indicated by equation (6).

$$CC_{ND,t} = k \cdot CC_{LD,t} \cdot (1 - \frac{CC_{LD,t}}{\overline{CC}_{MD}}) \tag{6}$$

Where $\dot{CC}_{LD,t}$ and $CC_{LD,t}$ indicate the change and current per capita consumption from less developed regions at time t, $\overline{CC}_{MD}$ the stable consumption level (or saturation point) in more developed regions and k represents the average growth rate. Additionally, demand in more developed countries needs to include changing effects driven by technology and substitution on the material composition of products (Tilton and Guzman 2016, ch. 2). Modelling those effects in the frontier is challenging considering the disruptive effect of technology and consumer preferences in the long-term. We propose to follow simple ARIMA models to model per capita consumption in more developed regions.

The second modification requires demand to adapt as we deplete known mineral stocks and prices increase. In the previous section, we model demand at any time as a function of income or population growth and substitution effort taken in the past (equation (4)). We explicitly consider substitution efforts as a function of past prices, where substitution efforts increase with an increase in prices ($\frac{\partial s}{\partial p_{t-1}} > 0$). Therefore, demand will evolve over time according to equation (7). The first term in equation (7) refers to the change in demand as a result of increasing income or population (IU hypothesis). The second term reflects the offsetting effect of substitution. The last term indicates that demand may suffer additional unpredictable shocks over time.

$$\frac{\partial D}{\partial t} = f_g \cdot \dot{g} + f_s \cdot \frac{\partial s}{\partial p_{t-1}} \cdot \frac{\partial p_{t-1}}{\partial t} + f_t \tag{7}$$

We expect that prices in the long-term are given by the CAC function as indicated in section 2.2. Then, we can obtain a more useful expression for $\frac{\partial p_{t-1}}{\partial t}$,

$$\frac{\partial p_{t-1}}{\partial t} = \frac{\partial CAC^{-1}\left(\sum_{t_0}^{t-1} D_j\right)}{\partial t} = \frac{D_{t-1}}{\frac{\partial CAC}{\partial p}\left(CAC^{-1}\left(\sum_{t_0}^{t-1} D_j\right)\right)} = \frac{D_{t-1}}{\frac{\partial CAC}{\partial p}(p_{t-1})}$$

$$\frac{p_{t-1}}{\frac{\partial CAC}{\partial p}(p_{t-1}) \cdot \frac{p_{t-1}}{D_{t-1}}} = \frac{p_{t-1}}{\varepsilon_{p,CAC}} \cdot \frac{D_{t-1}}{CAC}$$

Where $\varepsilon_{p,CAC}$ denotes the percentage change in cumulative resources because of one percentage change in prices. Replacing the previous result in equation (7) generates equation (8) below.

$$\frac{\partial D_t}{\partial t} = f_g \cdot \dot{g} + f_s \cdot \frac{\partial s}{\partial p_{t-1}} \cdot \frac{p_{t-1}}{\varepsilon_{p,CAC}} + f_t = f_g \cdot \dot{g} + f_s \cdot \frac{\varepsilon_{p,s} \cdot s \cdot \frac{D_{t-1}}{CAC}}{\varepsilon_{p,CAC}} + f_t \tag{8}$$

The term $\varepsilon_{p,s}$ represents the price elasticity of substitution efforts. From equation (8), substitution efforts become more important as we approach the inelastic section of the CAC and can dominate as mineral depletion increases over time. As discussed by Tilton and Guzman (2016, ch. 2), consumption may change in ways impossible to know due to substitution and changes in preferences. Material substitution in response to prices may take a decade or more to materialize, suggesting that price elasticity of substitution effort could be closer to zero in the medium-term (Smith and Eggert 2018). Nevertheless, we can obtain a conservative depletion measure from the CAC, if we omit substitution effects while prices are able to provide additional geological resources. As we deplete deposits, long-term prices increase, and we move to a more inelastic section of the CAC. When $\varepsilon_{p,CAC}$ becomes less than one $(\varepsilon_{p,CAC} < 1)$, there is little room to provide minerals from known sources and substitution effort should dominate. We can mark depletion at the point where $\varepsilon_{p,CAC}$ equals one, meaning that one percent increase in price generates less than a one percent increase in available resources.

3.2. Supply

The misconception of reserves and resources as fixed stocks is a common critique of peak models. Mineral resources are defined as a concentration of minerals in such form and amount that economic extraction of a commodity is currently or potentially feasible. In a higher technical and economic level, reserves are a part that could be economically extracted at the time of determination (U.S. Bureau of Mines and U.S. Geological Survey 1980). In this sense, if we take reserves and resources as fixed geological units, we will be mistakenly measuring future availability, not considering how they change with economic conditions (Crowson 2011, Meinert, Robinson and Nassar 2016). For example, peak models have been constantly adjusting their measures of the Ultimate Recoverable Resources (URR). Remaining copper stocks using URR estimates increase from around 1,400 million tonnes (Laherrére 2010), to 1,800 million tonnes (Northey, et al. 2014, Henckens, Driessen and Worrell 2014) more recently to 2,100 million tonnes (Calvo, Valero and Valero 2017). The changing URR is a major source of uncertainty in peak models, and their total stock estimates are usually too low (J. Tilton 2018).

The CAC reflects full extraction costs of all available deposits over all time. Equation (9) separates the CAC at any given time t in reserves ($Rv_t(p)$), resources ($Rc_t(p)$) and undiscovered deposits ($UD_t(p)$). These measures can change over time, but the CAC should remain as a stock measure. It is only possible to generate an imperfect CAC based on available information of known deposits.

$$CAC_t(p) = Rv_t(p) + Rc_t(p) + UD_t(p) \tag{9}$$

As indicated in equation (10), depletion analysis requires to acknowledge that information changes over time. At every period, reserves are depleted through consumption and a fraction of resources become reserves through mine planning and investment (α_t). Additionally, a fraction of undiscovered deposits generates reserves depending on exploration effort (β_t).

$$\frac{dRv_t}{dt} = -D_t + \alpha_t \cdot Rc_t + \beta_t \cdot UD_t \tag{10}$$

In any given time, most available information to construct the CAC comes from reserves, representing a higher certainty level about potential extraction. As reserves are depleted, prices increase, and resources become more attractive. Then, long-term depletion analysis should include resources as a future minerals source. Unfortunately, there is little research about reserves conversion over time (Northey, et al. 2014). If we do not have much information about resources, then we should penalize them giving them a higher extraction cost, to reflect the behavior described in equation (10). Assuming β_t as zero provides a conservative estimate for depletion. In this way, we focus on known deposits rather than unknown information. The modified CAC turns the available mineral stock under current information, extraction techniques and economic conditions. Assuming that current geological information accounts for all deposits in the earth's crust is misleading, but we also need to take advantage existing data to analyze mineral depletion. Modifying the CAC for different stocks (reserves, resources and undiscovered deposits) provides a first approach to include the effect of changing information over time. However, there is still enormous potential to support consumption with undiscovered deposits. For example, deep underground copper deposits are estimated in more than 40 times most optimistic URR measure (Kesler and Wilkinson 2008, Meinert, Robinson and Nassar 2016). Global copper resources have been continuously increasing, despite the increase in production and decline in ore grades (Mudd, Weng and Jowitt 2013). The increase in copper resources is not only driven by exploration, but also by economic, social, political and environmental considerations, rising concerns about using physical measures like tonnes or grades to conclude about future copper supply (Mudd and Jowitt 2018) .

4. Applying the Modified Cumulative Availability Curve to copper resources

A common or reconciled view of mineral depletion should combine the strong and simple depletion points of peak models and the economic scarcity created by increasing costs. In any economic analysis, supply and demand are defined simultaneously and relate to each other through the market price. However, as we have mentioned, long-term forecasts of supply and demand are greatly dependent on unpredictable variables. Thus, as recommended by Rosenau, et al. (2009), we propose different demand scenarios and assess how known deposits meet these scenarios.

4.1. Data sources

Previous literature on mineral depletion focuses on how copper demand increases as population grows (Northey, et al. 2014, Sverdrup, Ragnarsdottir and Koca 2014). Our population forecast by country is based on *World Population Prospects* considering constant-fertility and constant-mortality prepared by the Population Division of the Department of

Economic and Social Affairs in the United Nations (United Nations 2017). Total copper consumption per country from 1950 to 2017 comes from the historical archive of the Chilean Copper Commission (COCHILCO 2018).

Available copper deposits were divided into reserves, resources and an undiscovered deposits. Reserves data from countries are based on United States Geological Survey estimation (USGS 2018), accounting for 790 million tonnes in 2017. Current reserves and resources were estimated at 3,256 million tonnes based on Metals & Mining Properties database (S&P Global Market Intelligence 2017). Following Kesler and Wilkinson (2008), estimated recoverable underground copper deposits (known and unknown) are about 89,000 million tonnes. These undiscovered deposits are expected to have similar ore quality as known underground deposits, therefore can be extracted with existing methods.

Average extraction costs of reserves are based on Wood Mackenzie's Copper Mine Costs Workbook Q42016, considering total extraction costs (C3) for each country (Wood Mackenzie 2017). Average resources extraction costs were estimated for each country considering the last cost quartile (25% highest extraction costs) and using the highest world average quartile for those without mining information. Unknown and undiscovered deep underground deposits were estimated to have conditions similar to current high-cost underground operations. We consider the highest cost quartile from underground mines as the approximate extraction cost for undiscovered underground copper deposits.

4.2. Demand side

Per capita consumption in more developed regions is a non-stationary series (we fail to reject the null hypothesis of unit root at the 10% level) and our best estimate for future per capita consumption in more developed regions is 5.31 kg/person, based on an ARIMA (0,1,0) model[3]. To estimate future per capita consumption in less developed regions, we apply equation (6) with various stable consumption levels by distinguishing three main periods in more developed countries. First, a reconstruction period from 1950 to 1969. Then an industrialization and consumption period between 1970 to 1999. Finally, a digitalization period beginning in 2000. Figure 3 indicates that per capita copper consumption between 1970 and 1999 increased at every quantile with respect to consumption in the 1950 – 1969 period. For example, 50 percent of the population in more developed regions consumed on average less than 6.76 kg/person in the 1950 – 1969 period and the same percentage consumed less than 9.03 kg/person in the 1970 – 1999 period.

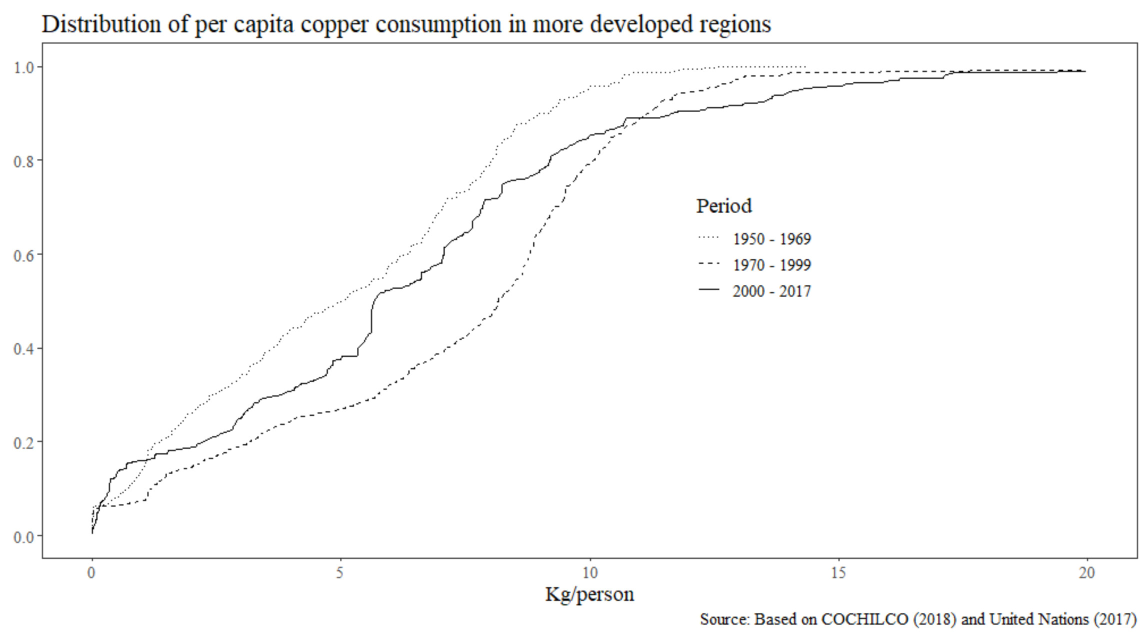

Figure 3 Per capita copper consumption in more developed regions, 1950 - 2017. Each curve indicates the distribution of per capita copper consumption in more developed regions in every period. The vertical axis represents the percentage of the population in more developed regions consuming less than the corresponding value in the horizontal axis. For example, 50 percent of the population in more developed regions consumed on average less than 6.76 kg/person in the 1950 – 1969 period

[3] The ARIMA (0,1,0) purges serial correlation in the residuals and it is preferred using information criteria (AIC and BIC) compared to ARIMA (1,1,0), ARIMA (0,1,1) and ARIMA (1,1,1).

We take advantage of the data in **Figure 3** to define four potential stable consumption levels. The 50th quantile of the first period, the 50th of the second period, the 40th quantile of the last period and the 60th quantile of the last period. These values represent a wide range of growth alternatives for less developed regions. Additionally, we include a scenario defined by the consumption forecast in more developed regions as indicated in **Table 1.**

Table 1 Scenarios of stable per capita copper consumption in less developed regions.

Scenario	Kg/person	Description
1	5.31	ARIMA forecast in more developed regions
2	5.46	40th quantile from the 2000 – 2017 period
3	6.76	50th quantile from the 1950 – 1969 period
4	7.06	60th quantile from the 2000 – 2017 period
5	9.03	50th quantile from the 1970 – 1999 period

Every scenario considers the 1950 consumption level as the required initial condition, and we perform maximum likelihood to calibrate the growth parameter that better fits historical data. **Figure 4** shows that models do not greatly differ to explain historical data, representing the past exponential behavior.

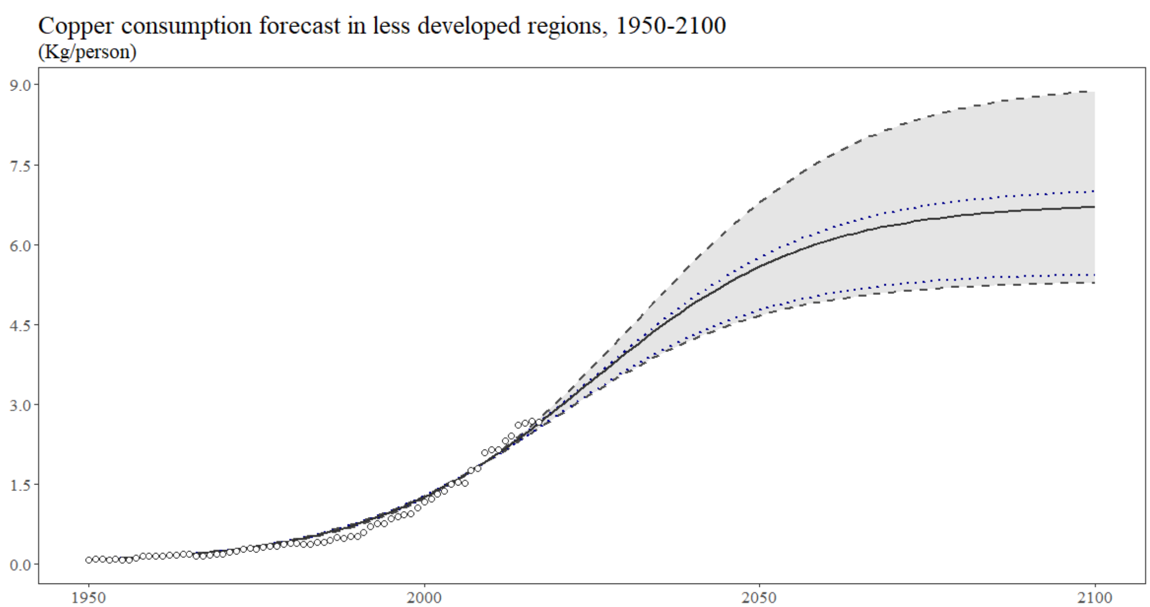

Figure 4 Per capita copper consumption in less developed regions, 1950-2100. Each line denotes a different stable consumption level from 5.31 to 9.03 kg/person. Every scenario is calibrated using maximum likelihood, varying the stable consumption level and defining

1950 consumption as the initial condition. The solid line shows the scenario with a stable level of 6.76 kg/person. Solid points represent historical per capita consumption.

Projected accumulated consumption from 2018 to 2100 appears in **Figure 5**. In 2050, accumulated total demand (including demand for recycled products) is expected to reach 1,193-1,551 million tonnes; and 4.788-7,136 million tonnes by 2100. Accumulated consumption from less developed regions represents 82 – 85 percent by 2050 and 89 – 93 percent by 2100. Recycling rate is assumed to remain fixed in each demand scenario at 14.5 percent, the average global value from 2013 to 2017 (World Metal Statistics 2018).

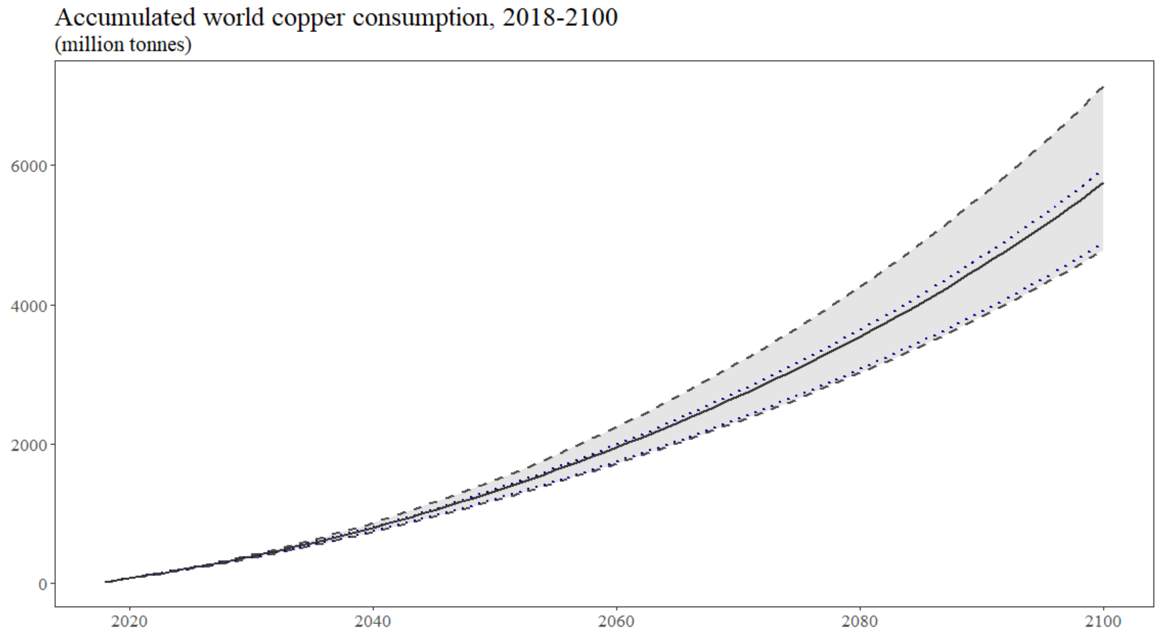

Figure 5 Accumulated world copper consumption, 2018 to 2100. Accumulated demand forecast adds up yearly refined total consumption starting from 2018. Different lines represent different growth scenarios for less developed regions. In more developed regions, every scenario considers a per capita consumption of 5.31 Kg/person.

4.3. Supply side

A common CAC represents remaining mineral tonnage in the horizontal axis and the expected cost to provide them on the vertical axis. In the modified cumulative availability curve (MCAC), we represent different supply sources according to their certainty level in reserves, resources and undiscovered deposits. Despite having different economic definitions, reserves and resources represent known copper deposits. As mentioned in section 4.1, we penalize resources by assigning them the top quartile in extraction cost from each country. The MCAC for reserves and resources is presented in **Figure 6**. The top figure illustrates that reserves are only a fraction of known deposits but are also expected to be mined first. Nevertheless, as we deplete reserves, prices should encourage extraction from other known available deposits in the long-run. Given the previous expected behavior, **Figure 6** b) represents a preferred MCAC for long-term availability from known deposits.

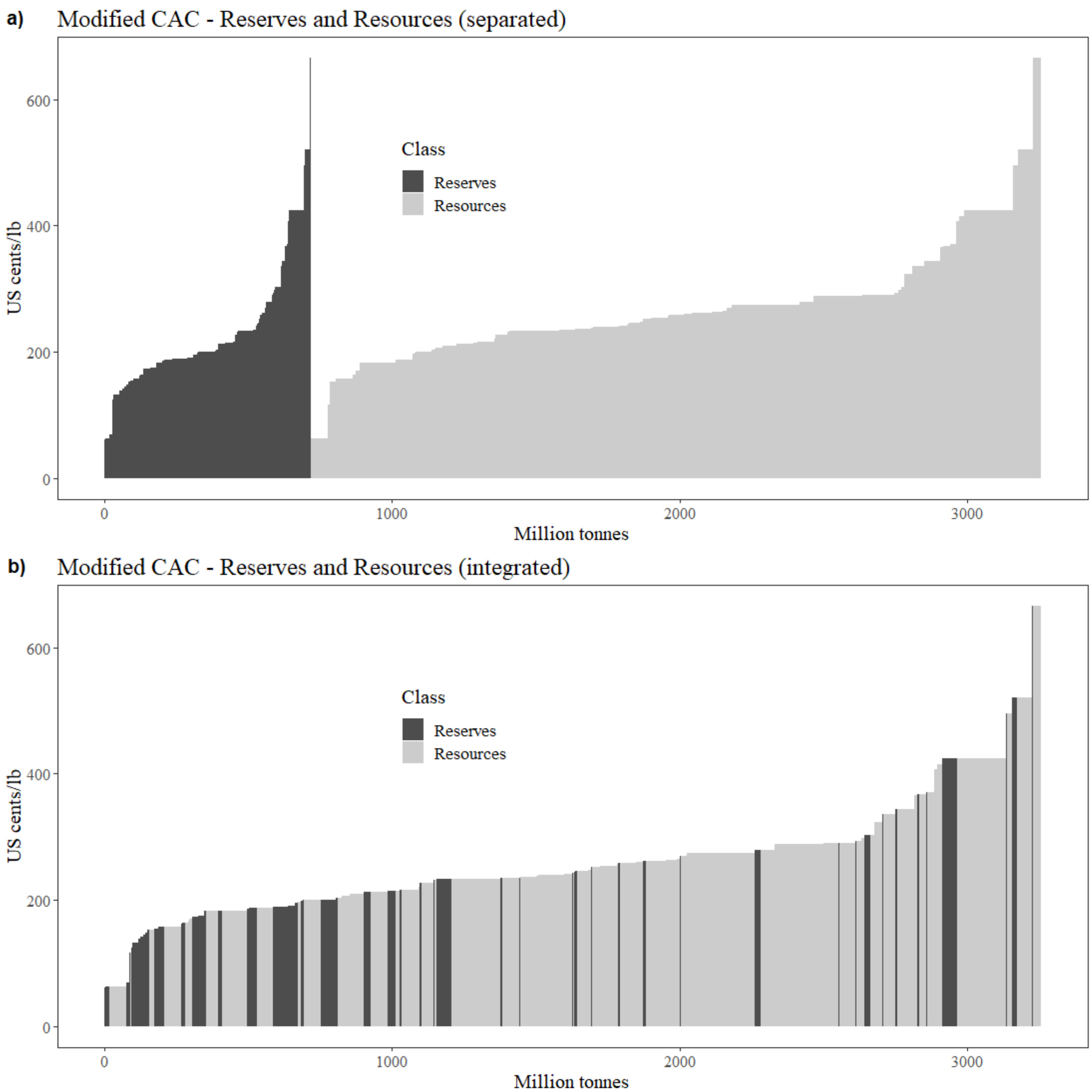

Figure 6 a) Modified CAC separating copper reserves and resources. b) Modified CAC integrating copper reserves and resources. The vertical axis in each graph indicates the estimated extraction cost from different countries based on cost quartiles. The horizontal axis indicates remaining contained copper from known deposits.

Underground unknown deposits are a potential source for minerals. Nevertheless, we lack confidence about their location and extraction costs. As previously stated, we assign the highest extraction cost quartile from current underground mines to undiscovered deposits. The assumption aims to provide a conservative measure of future mineral sources, given lack of information about discovery costs. Nonetheless, it is not clear that we can simply include undiscovered deposits in the MCAC with reserves and resources. The discovery barrier is marked by the vertical line in **Figure 7**. In practice, the figure serves two main purposes. First, it illustrates that undiscovered deposits can greatly contribute to minimize depletion if their geological characteristics are similar to known underground deposits. The shape of the curve suggests that we are unlikely to reach a rapid economic depletion but depends on the discovery rate. Second, the figure puts in perspective that physical depletion should not be a primary concern for copper. Instead, mineral resource management can benefit from expanding our current understanding of deep underground deposits and reserves conversion rate to avoid a discovery barrier.

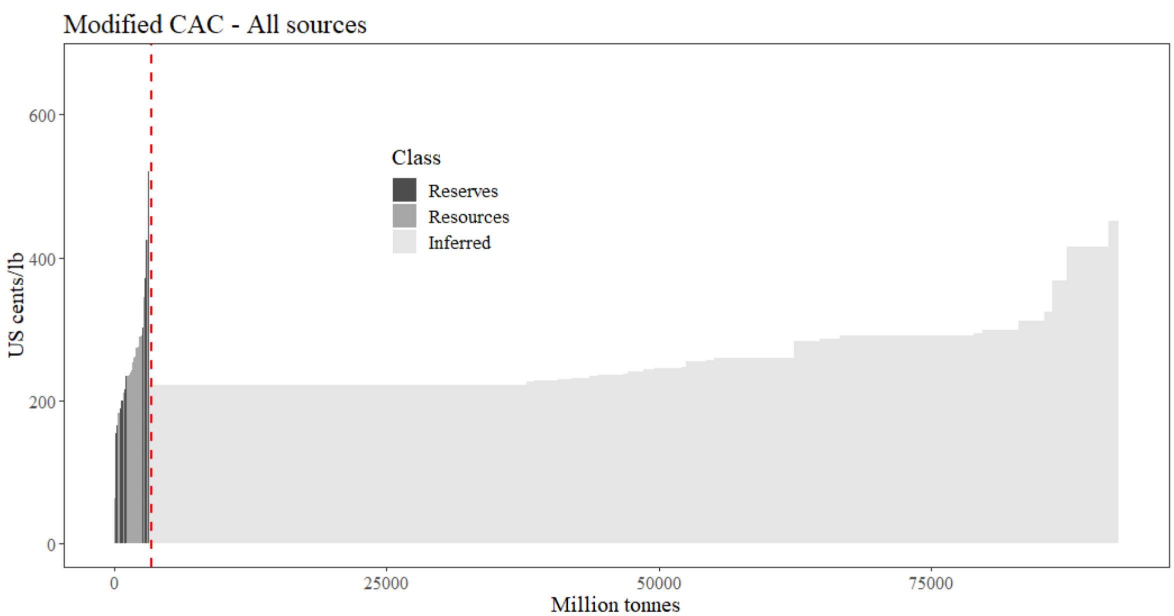

Figure 7 Modified CAC for reserves, resources and undiscovered deposits. The vertical axis in each graph indicates the estimated extraction cost from different countries based on cost quartiles. The horizontal axis indicates total remaining contained copper tonnage, separating known (reserves and resources) and unknown (undiscovered) deposits. The segmented vertical line indicates total current resource and reserves (3,256 million tonnes), representing not a physical barrier, but a discovery barrier.

4.4. Copper long-term availability

The modified CAC offers two main insights related to physical availability of known deposits and their economic scarcity. First, the availability from known deposits indicates that meeting demand by 2050 requires around one third of current reserves and resources, and around 80 percent by 2075 as shown in **Figure 8**. Nevertheless, the supply source composition is not homogenous across time. For example, in 2050, 64 percent of known reserves and 27 percent of known resources need to be extracted. After 2050 we observe a change in the slope of reserves consumption, indicating a heavier reliance on resources. This indicates that, in the medium to long term, the conversion rate from resources to reserves has a greater importance than physical depletion. In the most conservative demand growth scenario, current known deposits are insufficient only after 2075.

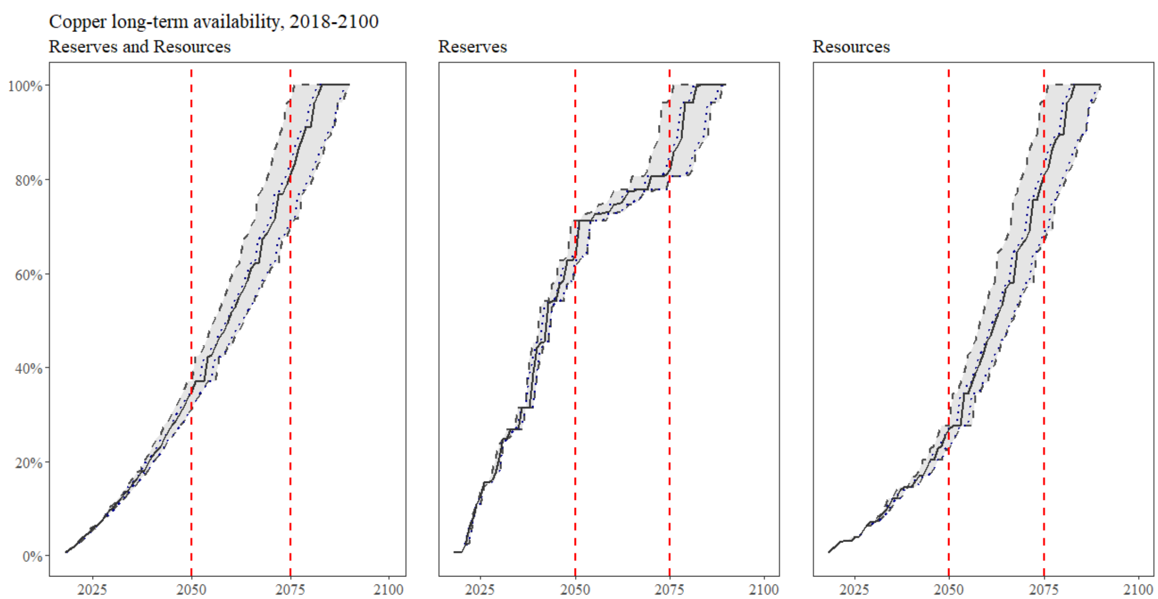

Figure 8 Copper extraction from reserves and resources, 2018-2100. The vertical axis indicates the percentage extracted over time for reserves and resources (left), reserves (middle) and resources (right) over time. Each figure illustrates different consumption scenarios. Vertical lines appear as references for years 2050 and 2080.

As demand increases in the long-term, deposits are depleted starting from those with lower costs. In this sense, economic scarcity appears as we move to the inelastic section of the modified CAC, indicating that an increase in price generates a less than proportional increase in quantity supplied. We approximate elasticity by fitting a polynomial function to the MCAC and calculating the price elasticity as the percentage change in quantity supplied every year as a response of a one percent change in price ($\frac{\%\Delta Q}{\%\Delta P}$). According to **Figure 9** and depending on the demand scenario, we would expect to achieve the inelastic part between 2069 and 2079. The result contrasts with depletion suggested by copper peak models, where supply fails to meet demand by 2033-2050. Additionally, the estimate from the MCAC is still conservative, because as we approach to the inelastic section, markets will create strong incentives to search for undiscovered deposits and substitute primary metal sources.

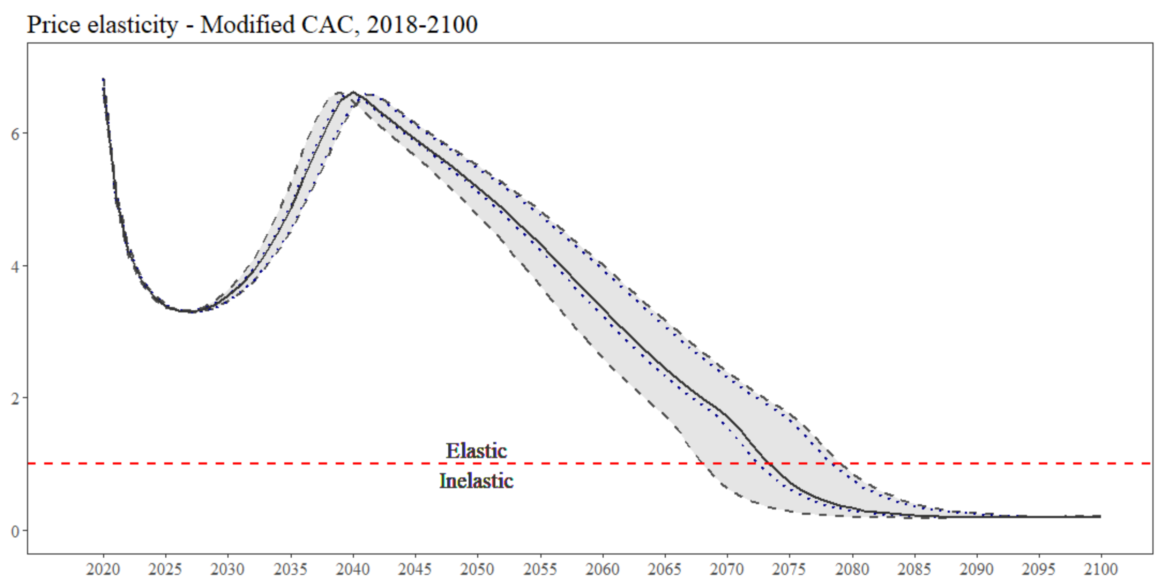

Figure 9 Price elasticity of the modified CAC for copper reserves and resources, 2018-2100. The vertical axis indicates the price elasticity of quantity supplied based on the modified CAC. Each line illustrates different consumption scenarios. The horizontal segmented line separates elastic and inelastic sections of the modified CAC.

5. Conclusions

Mineral depletion is one of the main debate topics in mineral resources management for sustainable development. Historically there have been two major approaches to assessing depletion: a fixed-stock paradigm and an opportunity-cost paradigm. The former is based on the self-evident fact that the earth is finite and so are the minerals contained in it, mostly relying on peak models to analyze mineral depletion. The latter supports that economic behavior may change long before actual physical depletion takes place; the analysis is not so much about the extraction of fixed stocks as it is about of what society needs to give up for them. The opportunity-cost paradigm prefers to analyze mineral depletion based on the cumulative availability curve (CAC).

In this paper, we review applications of peak models and the CAC, finding that both approaches consider that supply and extraction costs drive depletion in the long-term, and both acknowledge that geological information is at best uncertain. As a more general framework, we modify and formalize the cumulative availability curve, including different demand scenarios for less- and more-developed countries and considering uncertainty in geological information. In this way, we propose a depletion measure as we move to the inelastic (steeply increasing) part of the CAC, assuming that substitution efforts are mild before that point. These modifications consider major critiques elaborated by each paradigm.

Implementing the modified cumulative availability curve (MCAC) for copper suggests that current reserves and resources can meet world consumption until 2075 maintaining current recycling rates. This is a more optimistic view than peak models, where peak production should occur before 2050. Additionally, the MCAC model highlights that physical depletion is not the main issue when analyzing future copper supply. The conversion rate from resources to reserves is more important in the medium-term and deposit discovery rate is more important in the long-term.

The MCAC for copper does not indicate a future insurmountable barrier after 2075. Future mineral depletion depends on two main unknown variables requiring further research. First, the conversion rate from resources and undiscovered deposits to reserves represents an underdeveloped topic than can improve the dynamic aspect of the depletion analysis. Second, we are mostly unaware of society's ability to substitute metals in the long-term as a response to increases in prices. Both aspects, related to supply and demand for minerals, will improve MCAC outcomes and policy guidance about mineral sustainability.

Finally, in contrast to peak models, urging mineral-rich countries to take protective measures about their resources seems not advisable based on the copper MCAC. Delaying peak production can lock natural capital for a country, taking away the opportunity to support their economic growth. Additionally, restricting mineral production will only increase the threat of substitution, potentially destroying markets for a country's mineral assets. In this sense, research in new exploration techniques to find deep-covered deposits and in improving the efficiency of mineral processing and extractive metallurgy, as well as policy initiatives to incentivize exploration in technically-challenging and riskier deposits, are valuable to reduce impacts from resource depletion.

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