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Working Paper 2018-04
http://econbus-papers.mines.edu/working-papers/wp201804.pdf

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July 2018

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Working Paper No. **2018-04**
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Drill-bit parity: Supply-side links in oil and gas markets*

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ABSTRACT
Previous analyses of relationships between crude oil and natural gas markets focused primarily on demand-side connections. We provide a model and empirical evidence of important supply-side connections. First, crude oil and natural gas production require common inputs: drilling rigs, well completion services, and specialized labor. Competition for these inputs creates a *cost-spillover* channel through which a price shock for one commodity reduces supply of the other commodity. Second, crude oil wells produce associated gas, while natural gas wells often produce associated liquid hydrocarbons. This creates an *associated-commodity* channel through which a price shock for one commodity will increase supply of the other. Which effect dominates depends on the characteristics of the producing region. We test the model using well-level data from five large oil and gas producing basins in Texas and Oklahoma. We find substantial evidence across all five basins of a cost-spillover channel between natural gas prices and oil drilling, but mixed evidence of an associated-commodity channel between oil prices and natural gas drilling. Finally, we discuss the implications of these supply-side connections for energy policy.

JEL **classifications: D72, F18, F59, Q56**
Keywords: Energy, Petroleum, Crude Oil, Natural Gas

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1 Introduction

Following the financial crisis of 2008 and throughout the subsequent boom in shale oil production, U.S. crude oil and natural gas prices diverged substantially (see Figure 1). An empirical literature subsequently argued that the two markets are no longer linked in the way that they had been historically (Erdős, 2012; Ramberg and Parsons, 2012). Understanding the relationship between crude oil and natural gas markets is a fundamental problem for policy makers and investors. A policy or market shift that directly affects the market for one fuel will indirectly affect the other in a way that depends on how the markets interact. For example, the lifting of the crude oil export ban, the construction of liquefied natural gas export terminals, electricity market regulations that favor natural gas power plants, or changes in access to public lands for exploration and production in regions that are relatively more abundant in one fuel than the other.

Previous studies have generally appealed to demand-side connections because crude oil and natural gas are substitutes in the production of energy - the so-called "burner-tip parity" (Brown and Yücel, 2008; Hartley et al., 2008). This paper investigates supply-side connections between oil and gas markets. Supply-driven linkages are plausible for two reasons. First, drilling rigs, well completion services, and specialized labor are the primary inputs necessary for drilling crude oil wells and natural gas wells, so we might expect cost spillovers in the competition for inputs. Figure 2 displays time series of U.S. total active gas-directed drilling rigs and oil-directed drilling rigs from January 1997 to June 2016. From the figure, there appear to be periods of comovement and periods of divergence between the series. Second, joint production of natural gas from crude oil wells (sometimes called associated gas) is so commonplace that many analysts believe changes in oil-related drilling activity directly affect natural gas markets (Wall Street Journal, 2016; Bloomberg, 2015). Similarly, "wet" natural gas wells produce large amounts of liquid hydrocarbons such as butane and propane, which are close substitutes for the mix of hydrocarbons extracted from oil wells. These associated commodity flows could affect drilling rates for the opposing commodity; if a demand-shock increases the price of oil (gas), drilling for gas (oil) becomes more economically attractive if each new well also produces significant amounts of oil (gas). In this case, the cross-price supply elasticity is positive.

This paper derives a dynamic model of the drilling problem that clarifies the importance of each of these features in tying U.S. natural gas markets and crude oil markets through the supply side. We characterize the *cost-spillover* channel through which input competition affects supply, and the *associated-commodity* channel through which joint production affects the supply response to price shocks. The model incorporates the insights of Anderson et al. (2014) and Okullo et al. (2015) in that drilling new wells is the relevant margin for supply decisions. In our model, a representative firm makes continuous time decisions over drilling rates for each commodity. Drilling initiates new flow of the target commodity, but also of the opposite, or associated, commodity at a known rate. Once production is initiated from a new well, the flow of both the target and associated commodity is geologically constrained and declines at an exogenous rate as pressure in the well declines. Supply decisions are highly persistent as a result. We derive expressions for the cross-price drilling rate and supply responses of each commodity that depend on cost and geologic parameters. Our model applies to onshore crude oil and natural gas production in the United States, so we estimate the own- and cross-price drilling responses using well-level data from from five large oil and gas producing basins in Texas and Oklahoma.

We find strong evidence of the existence of supply-side links between oil and gas markets. Our evidence is consistent with the cost-spillover channel from gas price shocks to oil drilling across all five basins. We also find mixed evidence for an associated-commodity channel from oil price shocks to gas drilling. The relative strength of these cost-spillover and associated-commodity channels varies by basin. Our model implies that the sign of the cross-price drilling responses depends on the relative magnitudes of the opportunity cost of inputs versus the associated commodity flow rates. We also estimate the associated commodity flow parameters using well-level data from each basin and find significant heterogeneity across basins. We find that differences in our estimated cross-price drilling responses across basins are consistent with the regional variation in associated commodity flow parameters. For example, gas drilling responds most strongly to oil price shocks in the Anadarko Basin where associated oil is produced from gas wells at a relatively high rate.

Our findings have a variety of policy implications. Electricity policies that favor natural gas-fired power generation may have the indirect effect of reducing the supply of oil by raising the opportunity cost of its production. Examples include emissions rules that raise the cost of operating coal plants, the increasing need to manage intermittent renewable power sources, and a variety of carbon abatement policies. Similarly, if moves by OPEC to constrain oil supply are effective at raising the global price of oil then natural gas production in the United States will increase through the associated commodity channel. This would create a windfall for electricity producers, natural gas utility customers, and other end users. These tradeoffs is also important for setting carbon budgets, which are an increasingly popular tool for national governments to negotiate and implement international climate change agreements, for determining optimal resource taxes, and for understanding patterns of local economic impact from production. We expand on these implications in the discussion section.

The rest of the paper is organized as follows. Section 2 discusses the importance of supply-side links in the context of previous literature on oil and gas markets while section 3 formalizes these links in a model. Section 4 discusses the empirical approach and data, and section 5 presents the results. Section 6 discusses the results in terms of the policy implications, and section 7 concludes.

2 Background

Understanding the relationship between oil and natural gas markets is important for several reasons. The shale boom made the U.S. the largest producer of both crude oil and natural gas,[1] and helped dampen the 2007-2009 recession and expedite the subsequent recovery in shale-rich regions (Lim, 2011; Grunewald and Mahon, 2011; Brown and Yücel, 2013). A large literature documents the impact of oil price shocks on the U.S. economy.[2] More recently Hausman and Kellogg (2015) estimate shale-related production of natural gas created an annual increase of $48 billion in consumer and producer surplus, while Arora and Lieskovsky (2014) find that the impact of natural gas supply on industrial production has increased in the shale era. The increase in production of crude oil and natural gas has also significantly shifted environmental benefits and costs (Johnsen et al., 2016; Knittel et al., 2015; Linn et al., 2014; Muehlenbachs et al., 2013, 2015; Olmstead et al., 2013). Crude oil and natural gas are responsible for a large share of U.S. emissions of carbon dioxide and many other pollutants. Environmental or other policies that directly affect one commodity may indirectly distort the emissions rates of carbon and associated co-pollutants from the other commodity. The extent to which this occurs depends on the mechanisms by which the two markets

[1] See British Petroleum Statistical Review of World Energy (`http://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html`).

[2] See Balke et al. (2002) and Baumeister and Kilian (2016) for excellent surveys of this literature.

are linked.

A decline of demand-side substitution possibilities, and the observed divergence between oil and gas prices starting in 2008, led to a new line of research analyzing the instability of the direct statistical relationship between the two prices. See for example Erdős (2012), Ramberg and Parsons (2012), Aloui et al. (2014), Atil et al. (2014), Brigida (2014), Hartley and Medlock III (2014), although instabilities had been documented in some of the earlier literature that established the relationship in the first place, e.g., Villar and Joutz (2006), Brown and Yücel (2008) and Hartley et al. (2008). With the exception of Hartley and Medlock III (2014) who provide a model to clarify and test the role of exchange rates in determining the relative price of crude oil and natural gas this research is almost entirely based on reduced-form time series analyses of the prices. Such analyses may miss alternative mechanisms through which oil and gas markets are related. For example, the increase in onshore drilling for oil and natural gas in the continental U.S., using similar extraction techniques between the two commodities in adjacent locations, raises the need to analyze such alternative mechanisms.

There are three mechanisms that might create a link between crude oil and natural gas markets: demand-side substitution, supply-side input competition (e.g., rigs, well completion services, fracking materials, etc.), and supply-side joint production from associated commodity flows. Responses to price shocks will depend on the relative magnitudes of these effects. Under demand-side substitution, a positive oil price shock would lead consumers to shift their consumption toward natural gas, for example by increasing their use of natural gas relative to oil in electricity generation. This in turn would lead to a higher gas price, and increased gas drilling. A positive natural gas price shock would lead to a higher oil price, and increased oil drilling for the same reasons.

Under supply-side input competition, a positive oil price shock would lead to increased oil drilling, which would increase the marginal cost of natural gas drilling, and decrease natural gas drilling, as our model will show. Similarly, a positive natural gas price shock would lead to decreased oil drilling. Contrarily, if the associated commodity channel is dominant, i.e., oil wells produce significant marketable quantities of natural gas, and/or natural gas wells produce marketable quantities of liquids, then it is possible that a positive natural gas price shock will lead to increased oil drilling, and a positive oil price shock will lead to increased gas drilling, as the joint production makes the marginal well more profitable. Our empirical section investigates which of these factors dominates at any given time and place.

3 Model

Consider a social planner who maximizes total surplus in the economy by initiating flows of oil and natural gas from available reservoirs. Let the state variables $z_o(t)$ and $z_g(t)$ represent the flow of crude oil and natural gas from these reservoirs at time t. The history of drilling activity determines the current flow of output $z_o(t)$ and $z_g(t)$ as follows. At each instant, the planner invests in increasing the flow of each commodity by amounts $q_o(t)$ and $q_g(t)$. In practice, these represent all the steps required to drill and complete a new well in order to bring new flows to market, including allocating drilling rigs, active drilling, hydraulic fracturing, allocating well completion teams, etc. In this paper, we will use the terms "drilling" or "rig allocations" as a shorthand for the set of actions by which firms in practice bring new flow of crude oil and natural gas online from existing reservoirs. In our continuous time representation of this process, these actions are controlled smoothly at each instant t through the choice variables $q_o(t)$ and $q_g(t)$.

The flow of production from previously drilled oil and natural gas wells, $z_o(t)$ and $z_g(t)$, declines over time due to geological characteristics of the underlying reservoir (Cronquist, 2001).[3] For our purposes, we assume that the geological rate of decline is constant across wells and through time within a well, or that there is a constant exponential decline rate, which may differ between crude oil and natural gas.[4] Let α_o denote the decline rate for oil wells, and let α_g denote the decline rate for natural gas wells.

We assume the increase in gas flow associated with oil drilling, and the increase in oil flow associated with gas drilling can be characterized by proportions, ψ_g and ψ_o, of the respective drilling levels, where ψ_g and ψ_o are in [0,1). That is, when oil flow increases by $q_o(t)$ through drilling, gas flow also increases by $\psi_g q_o(t)$, and vice versa. We can now write the dynamics of crude oil and natural gas production as

$$\dot{z}_o(t) = q_o(t) - \alpha_o z_o(t) + \psi_o q_g(t) \tag{1}$$

and

$$\dot{z}_g(t) = q_g(t) - \alpha_g z_g(t) + \psi_g q_o(t).^5 \tag{2}$$

[3]In Anderson et al. (2014) the state variable is production flow capacity, and actual production flow is a control variable. However, those authors also show that the production flow will be equal to production flow capacity unless there is a steep contango in the commodity market, which is historically rare, so we simplify the model by implicitly assuming that flow capacity is always constrained.

[4]We make this simplifying assumption for tractability: natural gas and crude oil wells often exhibit hyperbolic decline, implying that the decline rate decreases as the producing life of the well increases. However, exponential decline offers a reasonable approximation for our purposes.

[5]The firm's total production of gas at t is

$$z_g(t) = \int_0^t [q_g(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds.$$

Using Leibniz Rule to take the time derivative gives

$$\dot{z}_g(t) = -\alpha \int_0^t [q_o(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds + q_g(t) + \psi_g q_o(t) = -\alpha z_g(t) + q_g(t) + \psi_g q_o(t).$$

Equations (1) and (2) indicate that production of the commodities increases with drilling rates and decreases due to geological decline. The exponential-decline assumption leads production to decline by a constant fraction of total production. Drilling also increases production of oil and natural gas through associated commodity flows, which are captured by the last terms on the right-hand sides of (1) and (2).

Oil and natural gas wells are drilled into scarce reservoirs. The availability of drilling prospects decreases as more drilling occurs. We represent the stock of drilling prospects available for crude oil at t by the continuous variable $A_o(t)$, while the stock of drilling prospects available for natural gas is $A_g(t)$, with initial drilling-prospect stocks $A_o(0) = A_{o0}$ and $A_g(0) = A_{g0}$ given exogenously. The dynamics of drilling prospect availability for oil and gas are then

$$\dot{A}_o(t) = -q_o(t) \tag{3}$$

and

$$\dot{A}_g(t) = -q_g(t). \tag{4}$$

It is costly to drill oil and natural gas wells. We assume that drilling costs for each commodity can be broken into three components: costs that are common to drilling for both commodities, costs that are commodity specific, and costs related to gathering associated-commodity flows. For example, in practice a drilling rig must be allocated whether oil or gas is being drilled for, so the cost of allocating a rig is common to both commodities. Similarly, well-completion services including specialized labor and materials represent scarce common inputs to drilling oil and natural gas wells. When these inputs are used to drill a well of one commodity type, the increased opportunity cost of inputs spills over into drilling for the other commodity type. We represent such costs that are common to both commodities by $C(q_o(t) + q_g(t))$, which we assume to be increasing and convex in total drilling.

Each commodity also has costs of initiating production that are unique to that commodity. These costs are often associated with takeaway capacity. For example, a new gas well will require pipeline capacity purchases, and may even require the construction of new pipelines, whereas oil from a new oil well can be taken away by truck or other transportation mode which may need to be expanded. The portions of cost that are commodity specific are assumed to be additively separable, and are denoted $C_o(q_o(t))$ for crude oil, and $C_g(q_g(t))$ for natural gas. We assume that these cost functions are increasing and convex in their respective drilling intensities, which represents scarcity of inputs involved in building out takeaway capacity, and other infrastructure necessary for initiating new streams of production.

The third components of drilling costs are related to associated-commodity flows and are denoted $C_{\psi_o}(\psi_o q_g)$ for associated oil and $C_{\psi_g}(\psi_g q_g)$ for associated gas. These costs are separated from the commodity-specific flow costs in the model, because collecting associated gas from oil wells may differ in fundamental ways from gathering natural gas from gas wells. For example, gas wells are more likely to be near pre-existing natural gas pipelines than oil wells. We assume these functions are increasing and convex, and that $C_{\psi_o}(0) = C_{\psi_g}(0) = 0$.

Consumer surplus associated with oil and gas is increasing in oil and gas flows, $z_o(t)$ and $z_g(t)$, and is also increasing in exogenous demand shocks, $s_o(t)$ and $s_g(t)$. For example, cold weather increases consumer surplus associated with a fixed supply of natural gas. Similarly, an OPEC production cut would increase consumer surplus associated with U.S. oil production – our geographic market of interest. Let $U_g(z_g(t), s_g(t))$ denote consumer surplus associated with natural gas flow at time t, and let $U_o(z_o(t), s_o(t))$ represent consumer surplus associated with oil flow at time t. We assume consumer surplus associated with each commodity is increasing and concave in flows. The marginal consumer surplus associated with each commodity will be the price of that commodity, so $\frac{\partial U_g(z_g(t), s_g(t))}{\partial z_g(t)} = P_g(z_g(t), s_g(t))$ and $\frac{\partial U_o(z_o(t), s_o(t))}{\partial z_o(t)} = P_o(z_o(t), s_o(t))$. Further, we assume that demand shocks increase the marginal value of oil and natural gas, or $\frac{\partial P_o(z_o(t), s_o(t))}{\partial s_o(t)} > 0$ and $\frac{\partial P_g(z_g(t), s_g(t))}{\partial s_g(t)} > 0$.

We also assume that total consumer surplus is the sum of surplus from oil and gas consumption. The social planner will maximize consumer surplus less production costs, so the social planner's problem can be written (with time dependence suppressed)

$$
\begin{aligned}
\max_{q_o, q_g} \quad & \int_{t=0}^{\infty} \left[U_o(z_o, s_o) + U_g(z_g, s_g) - C(q_o + q_g) - C_o(q_o) - C_g(q_g) - C_{\psi_o}(\psi_o q_g) - C_{\psi_g}(\psi_g q_o)\right] e^{-rt} dt \\
\text{subject to} \quad & \dot{z}_o = q_o - \alpha_o z_o + \psi_o q_g, \\
& \dot{z}_g = q_g - \alpha_g z_g + \psi_g q_o, \\
& \dot{A}_o = -q_o, \; A_{o0} \text{ given}, \\
\text{and} \quad & \dot{A}_g = -q_g, \; A_{g0} \text{ given},
\end{aligned}
\tag{5}
$$

where r is the discount rate. The current-value Hamiltonian associated with (5) is

$$
\begin{aligned}
\mathcal{H} = {} & U_o(z_o, s_o) + U_g(z_g, s_g) - C(q_o + q_g) - C_o(q_o) - C_g(q_g) - C_{\psi_o}(\psi_o q_g) - C_{\psi_g}(\psi_g q_o) \\
& - \theta_o q_o - \theta_g q_g + \mu_o(q_o - \alpha_o z_o + \psi_o q_g) + \mu_g(q_g - \alpha_g z_g + \psi_g q_o)
\end{aligned}
\tag{6}
$$

The θ_o and θ_g variables are the shadow values of scarce oil and gas drilling prospects, and represent the opportunity cost associated with using up the marginal drilling prospect at the current instant. The marginal value of initiating a new unit of commodity flow through drilling is greater than the price in this model, because a marginal unit of flow added in the current instant represents a stream of future production. The present values of these production streams are μ_o and μ_g, the costate variables on the flow equations.

Assuming an interior solution for drilling in both commodities, the first-order necessary conditions for surplus-maximizing rig allocations associated with (5) are

$$-C'(q_o + q_g) - C_o'(q_o) - \psi_g C_{\psi_g}'(\psi_g q_o) - \theta_o + \mu_o + \psi_g \mu_g = 0, \tag{7}$$

$$-C'(q_o + q_g) - C_g'(q_g) - \psi_o C_{\psi_o}'(\psi_o q_g) - \theta_g + \mu_g + \psi_o \mu_o = 0, \tag{8}$$

$$-P_o(z_o, s_o) + \alpha_o \mu_o = \dot{\mu}_o - r\mu_o, \tag{9}$$

$$-P_g(z_g, s_g) + \alpha_g \mu_g = \dot{\mu}_g - r\mu_g, \tag{10}$$

$$\dot{\theta}_o - r\theta_o = 0, \tag{11}$$

and

$$\dot{\theta}_g - r\theta_g = 0. \tag{12}$$

The following transversality conditions are also necessary for the optimality of a drilling program

$$\lim_{t \to \infty} \theta_o(t) A_o(t) = 0, \tag{13}$$

$$\lim_{t \to \infty} \theta_g(t) A_g(t) = 0, \tag{14}$$

$$\lim_{t \to \infty} \mu_o(t) z_o(t) = 0, \tag{15}$$

and

$$\lim_{t \to \infty} \mu_g(t) z_g(t) = 0. \tag{16}$$

3.1 Interpretation of Necessary Conditions

Equations (7) and (8) balance the current marginal costs of rig allocation with the present value of the marginal benefits of initiating new commodity flow. The first four terms on the left-hand sides of (7) and (8) represent the marginal costs of rig allocation. The first term in each equation, $C'(q_o + q_g)$ is the marginal cost of allocating an additional rig regardless of commodity choice. The second term in each equation is the commodity-specific marginal cost, which represents the increase in cost from initiating a unit of new flow of the target commodity. The marginal costs from increased associated commodity flows are represented by the third terms in (7) and (8). The initiation of a marginal unit of oil flow will be accompanied by an increase in natural gas flow of size ψ_g. This increase in natural gas flow will increase costs by $\psi_g C'_{\psi_g}(\psi_g q_o)$. Oil-drilling costs will similarly be increased via associated oil from natural gas wells. The fourth terms in (7) and (8) represent the user costs associated with converting the marginal drilling prospect into commodity flow. These costs derive from the fact that drilling prospects are exhaustible resources.

The last two terms in the left-hand sides of (7) and (8) represent the marginal benefits associated with initiating marginal units of oil and gas flow through rig-allocation decisions. For example, μ_o is the marginal benefit of having an additional unit of oil flow at time t by definition. Further, the natural gas flow will increase by ψ_g when an additional unit of oil flow is initiated, and $\psi_g \mu_g$ is the marginal benefit of this additional associated-gas flow. In order for maximum surplus to be achieved through strictly positive rig-allocations, it is necessary that the marginal costs and marginal benefits described here sum to zero.

Equations (9) and (10) define the inter-temporal marginal value of oil flow and natural gas flow. A unit of oil or gas flow at time t implies an infinite stream of future production whose value will decrease at instantaneous rate r due to discounting. The amount of flow will decrease at instantaneous rate α_i, for $i = \{o, g\}$ due to the natural geologic decline in production. Thus, production flow is discounted at instantaneous rate $r + \alpha_i$ when discounting and production decline are both taken into account. Equations (9) and (10) can be rearranged into the form $\mu_i = (P_i(z_i, s_i) + \dot{\mu}_i)/(\alpha_i + r)$. The value of a marginal unit of commodity flow is similar to the value of a perpetuity due to the associated infinite stream of production. Inter-temporal changes in the value of flow may occur through changes in demand, or changes in the scarcity of drilling prospects represented by $\dot{\mu}_i$.

Equations (11) and (12) imply that drilling prospects should be managed in such a way that their value increases at the discount rate, so that drilling prospects are competitive with other assets in the economy.

The transversality conditions represented in (13) and (14) show that drilling prospects must be completely depleted, or have zero marginal value in the long run. In the case that a finite number of drilling prospects are available, the combination of (11) and (13) imply that oil drilling prospects will be completely depleted in the long run, while the combination of (12) and (14) imply that gas drilling prospects will be completely depleted in the long run. The depletion of oil or gas drilling prospects imply that (15) and (16) will hold, as after the last drilling prospects are converted through drilling, production will decrease at the constant exponential decline rates and approach zero in the limit.

3.2 Infinite Well Model Results

In order to maintain focus on supply-side links between gas and oil markets in this paper, we analyze the "infinite well" case.[6] In this case, the social planner (or competitive market) behaves as though there is an infinite quantity of oil and gas drilling prospects available for conversion to production through drilling. This need not mean that market participants actually believe drilling prospects are infinite, but rather could mean that market participants believe lower-cost backstop technologies will become available prior to depletion of oil and gas drilling prospects. The assumption of infinite drilling prospect availability implies that $\theta_o(t)$ and $\theta_g(t)$ must go to zero in the limit by (13) and (14), which in conjunction with (11) and (12) implies that

$$\theta_o(t) = \theta_g(t) = \dot{\theta}_o(t) = \dot{\theta}_g(t) = 0 \quad \text{for all} \quad t. \tag{17}$$

The infinite drilling prospect assumption leads to the possibility of achieving a steady-state in which drilling rate, production rates, and shadow values remain constant. In particular, in a steady-state, we have $\dot{\mu}_o = \dot{\mu}_g = 0$, and we can rewrite (9) and (10) as

$$\mu_o = \frac{P_o(z_o, s_o)}{\alpha_o + r} \tag{18}$$

and

$$\mu_g = \frac{P_g(z_g, s_g)}{\alpha_g + r}. \tag{19}$$

In a steady state, the value of a marginal unit of production flow is exactly the value of a perpetuity discounted at the sum of the discount rate and the commodity-specific decline rate. Equations (18) and (19) can be substituted into (7) and (8) to get a full accounting of the marginal benefits and

[6] See Anderson et al. (2014).

marginal costs of drilling decisions in the steady state. We have

$$\underbrace{\frac{P_o(z_o, s_o)}{\alpha_o + r} + \frac{\psi_g P_g(z_g, s_g)}{\alpha_g + r}}_{\text{Marginal Benefit } (q_o\uparrow)} - \underbrace{C'(q_o + q_g) - C'_o(q_o) - \psi_g C'_{\psi_g}(\psi_g q_o)}_{\text{Marginal Cost } (q_o\uparrow)} = 0 \tag{20}$$

$$\underbrace{\frac{P_g(z_g, s_g)}{\alpha_g + r} + \frac{\psi_o P_o(z_o, s_o)}{\alpha_o + r}}_{\text{Marginal Benefit } (q_g\uparrow)} - \underbrace{C'(q_o + q_g) - C'_g(q_g) - \psi_o C'_{\psi_o}(\psi_o q_g)}_{\text{Marginal Cost } (q_g\uparrow)} = 0. \tag{21}$$

Equations (20) and (21) show that the planner will balance marginal benefits and marginal costs in the steady-state of an infinite-well optimal drilling program. The first terms in (20) and (21) represent the direct marginal benefit of initiating a marginal unit of oil or gas flow through drilling. This marginal benefit is the initiation of a perpetuity with payoff $P_o(z_o, s_o)$ or $P_g(z_g, s_g)$ per instant. The perpetuity is discounted at $r + \alpha_o$ or $r + \alpha_g$ where the latter terms result from the natural decline in production from oil or gas wells.

The initiation of a marginal unit of oil or gas production through drilling is accompanied by the initiation of ψ_g units of gas production or ψ_o units of oil production through the associated commodity channel. The second terms in equations (20) and (21) account for the marginal perpetuity value from associated commodity flows. The initiation of associated commodity flows will result in infinite payments similar to those of the direct commodity flow, however, this marginal benefit is weighed by ψ_g or ψ_o, the proportion at which the associated commodity is produced.

The third terms in (20) and (21) represent the marginal cost of allocating additional units of drilling inputs, irrespective of commodity. Costs will increase for both commodities with additional drilling for either commodity. The fourth terms in (20) and (21) represent the increase in commodity-specific costs associated with drilling for oil or gas, and the fifth terms represent the increase in costs that takes place as a result of initiating associated commodity flows. These last costs are weighed by the associated commodity parameters, ψ_g and ψ_o.

We can substitute the z_o and z_g terms out of (20) and (21) under the assumption that drilling has converged to an infinite-well steady state. In an infinite-well steady state, we have $\dot{z}_o = \dot{z}_g = 0$, substituting these values into (1) and (2), and solving for z_o and z_g gives

$$z_o = \frac{q_o + \psi_o q_g}{\alpha_o} \tag{22}$$

and

$$z_g = \frac{q_g + \psi_g q_o}{\alpha_g}. \tag{23}$$

Substituting these steady-state production values into (20) and (21) gives

$$f_o : \frac{P_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o + r} + \frac{\psi_g P_g\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g + r} - C'(q_o + q_g) - C_o'(q_o) - \psi_g C_{\psi_g}'(\psi_g q_o) \equiv 0 \tag{24}$$

and

$$f_g : \frac{P_g\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g + r} + \frac{\psi_o P_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o + r} - C'(q_o + q_g) - C_g'(q_g) - \psi_o C_{\psi_o}'(\psi_o q_g) \equiv 0 \tag{25}$$

When evaluated on the optimal drilling paths, equations (24) and (25) are two identities in two choice variables, q_o and q_g, defined at each instant t.[7] Thus, the implicit-function theorem can be applied in order to predict optimal drilling responses to parameter changes. The comparative statics of primary interest are the optimal steady-state drilling responses to own-commodity and cross-commodity demand shocks, i.e., the changes in optimal oil drilling rates in response to gas-market demand shocks and vice versa. In the Mathematical Appendix, we show that

$$\frac{\partial q_g}{\partial s_g} = \frac{\frac{-P_g^s}{\alpha_g+r}\left[\frac{(1-\psi_g\psi_o)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]}{|H|}, \tag{26}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\frac{-P_o^s}{\alpha_o+r}\left[\frac{(1-\psi_g\psi_o)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' - C_g'' - \psi_o^2 C_{\psi_o}''\right]}{|H|}, \tag{27}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o+r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]}{|H|} \tag{28}$$

[7]The Mathematical Appendix section also confirms that under the assumptions of the model, the identities (24) and (25) characterize a maximum solution to the planner's problem.

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g+r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]}{|H|}. \tag{29}$$

where $|H| > 0$ is the determinant of the Hessian matrix associated with (24) and (25) and $P_i^s = \frac{\partial P_i}{\partial s_i}$ represents a positive demand shock for commodity i. The own-price derivatives, defined in equations (26) and (27), are positive as expected as a result of the concavity of the utility function and convexity of the cost functions, while the signs of the cross-price derivatives, defined in (28) and (29), are indeterminate and will depend on the magnitudes of the associated-commodity parameters, and the degree of convexity of the various drilling cost functions. Note that if $C'' = \psi_g = \psi_o = 0$, then there is no input competition or associated commodity production, the cross-price derivatives evaluate to zero, and supply-side links are totally absent.

We define a *cost-spillover regime* as a regime in which cost spillovers, articulated in the model with the $C(q_o + q_g)$ function, play an important role in oil and gas markets, while associated-commodity flows play an arbitrarily small role. In such a regime, we have $C'' > 0$ and $\psi_g = \psi_o = 0$. Similarly, we define an *associated-commodity regime* as a regime in which associated-commodity flows play an important role in oil and gas markets, while cost spillovers play an arbitrarily small role. In such a regime, we have $\psi_g > 0$ and/or $\psi_o > 0$, and $C'' = 0$. These definitions lead us to two testable propositions.

Proposition 1: *In a cost-spillover regime, a positive oil price shock decreases the steady-state rate of natural gas drilling, and a positive natural gas price shock decreases the steady-state rate of oil drilling.*

Proof. If we substitute $\psi_g = \psi_o = 0$ from the definition of a cost-spillover regime into the comparative statics defined by equations (28) and (29), we have

$$\frac{\partial q_g}{\partial s_o} = -\frac{P_o^s C''}{\alpha_o + r} \Big/ |H| < 0 \tag{30}$$

and

$$\frac{\partial q_o}{\partial s_g} = -\frac{P_g^s C''}{\alpha_g + r} \Big/ |H| < 0. \tag{31}$$

The signs of the derivatives defined by equations (30) and (31) result from the assumption that $C'' > 0$ in a cost-spillover regime. □

Proposition 1 indicates that we should expect drilling rates to diverge when a positive gas or oil price shock occurs if supply-side links between oil and gas markets are primarily characterized by drilling-cost spillovers.

In an associated-commodity regime, on the other hand, a positive oil price shock has an ambiguous effect on the steady-state rate of natural gas drilling because there are two opposing effects. Increased oil drilling in response to the oil price shock will increase the supply of associated gas, reducing the incentive to drill for natural gas. By contrast, associated oil produced from gas wells will be more valuable, increasing the incentive to drill for natural gas. The situation is analogous in the case of a natural-gas price shock. We present Proposition 2 in two parts based on which of these effects dominates.

Proposition 2a: *In an associated-commodity regime, if*

$$\left| \frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g + r)} \right| > \psi_o \left[C_o'' + \psi_g^2 C_{\psi_g}'' \right] \tag{32}$$

then a positive oil price shock leads to a decrease in the steady-state rate of natural gas drilling, and if

$$\left| \frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o + r)} \right| > \psi_g \left[C_g'' + \psi_o^2 C_{\psi_o}'' \right] \tag{33}$$

then a positive natural gas price shock leads to a decrease in the steady-state rate of oil drilling.

Proof. If we substitute $C'' = 0$ from the definition of an associated-commodity regime into the comparative statics defined by equations (28) and (29), we have

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o + r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} + \psi_o C_o'' + \psi_o \psi_g^2 C_{\psi_g}''\right]}{|H|} \tag{34}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g + r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} + \psi_g C_g'' + \psi_g \psi_o^2 C_{\psi_o}''\right]}{|H|}. \tag{35}$$

The terms in the brackets on the right-hand sides of (34) and (35) are negative by (32) and (33). Therefore, under the assumptions of Proposition 2a cross-price drilling rate responses are negative. □

Proposition 2b: *In an associated-commodity regime, if*

$$\left|\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)}\right| < \psi_o\left[C_o'' + \psi_g^2 C_{\psi_g}''\right] \tag{36}$$

then a positive oil price shock leads to an increase in the steady-state rate of natural gas drilling, and if

$$\left|\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)}\right| < \psi_g\left[C_g'' + \psi_o^2 C_{\psi_o}''\right] \tag{37}$$

then a positive natural gas price shock leads to an increase in the steady-state rate of oil drilling.

Proof. The proof of Proposition 2b follows directly from the proof of Proposition 2a. □

Proposition 2 indicates that in an associated-commodity regime, the cross-price drilling response depends on the relative magnitude of the demand slope for the associated commodity versus the increase in drilling costs. The term in brackets to the left of the inequality represents the perpetuity value of a marginal decline in the associated commodity's price driven by new associated commodity flows. The term in brackets to the right of the inequality represents the increase in marginal drilling costs required to bring both new target flows and new associated commodity flows to market. Weighting the cost terms in brackets by the associated commodity parameter represents the opportunity cost of obtaining the target commodity from target wells versus obtaining it as an associated commodity from the opposite well type.

Consider the inequalities in (32) and (36), for example. Intuitively, an oil price shock will increase the steady-state oil drilling rate (see equation (27)), and will lead to increased associated-gas flow, thus decreasing the price of gas and disincentivising gas drilling. If associated gas from new oil drilling drives the price of natural gas down by a “wide enough” margin, then there is a decrease in the steady state rate of natural gas drilling (equation (32) holds). Here, “wide enough” is relative to a modest increase in the marginal cost to produce and sell the oil and associated gas and/or a modest increase in associated oil from gas wells.

On the other hand, associated-oil flows from gas wells will be more valuable as a result of the oil price shock. If the effect of associated gas from oil wells on the gas price is small relative to the production of associated oil from gas wells (ψ_o) along with the cost increase required to initiate new oil wells (equation (36) holds) then producers will respond to the oil price shock by increasing gas drilling and marketing the associated oil. We estimate these cross-price drilling responses in the empirical section below.

4 Empirical Approach

In this section, we describe our approach to estimating the cross-price drilling elasticities in five large crude oil and natural gas producing basins in Oklahoma and Texas in order to establish the supply-side links proposed by our model. We estimate a series of equations of the form

$$\ln q_{ibt} = \beta_0 + \beta_1 \ln q_{ib,t-1} + \beta_{ii} \ln P_{it} + \beta_{ij} \ln P_{jt} + \epsilon_{ibt} \tag{38}$$

where $\ln q_{ibt}$ is the natural log of new wells completed targeting commodity $i \in \{oil, gas\}$ in basin b in month t, and $\ln P_{it}$ is the natural log of the benchmark price of commodity i in month t, either the Henry Hub natural gas price or the West Texas Intermediate (WTI) crude oil price. We use the natural log of new wells as the dependent variable because drilling represents the marginal choice made by oil and gas producers as shown by Anderson et al. (2014) and Mason and Roberts (2018). As a robustness check, we also report results using the natural log of the initial peak quantity of the target commodity from new wells as the dependent variable.

The prices in equation (38) are likely to be endogenous despite the fact that oil markets are global and the Henry Hub is the most liquid U.S. trading hub for natural gas. The five basins used in our estimation are the Anadarko Basin (OK and TX), the Chautauqua Platform (OK), the East Texas Basin (TX), the Fort Worth Basin (TX), and the Permian Basin (TX). All of these basins are relatively close geographically to the Henry Hub and WTI trading points, and produce nontrivial quantities of crude oil and natural gas which could affect the prices.

We therefore use four types of instruments for crude oil and natural gas prices. Roberts and Schlenker (2013) suggest using lagged shocks to storage levels to identify supply and demand parameters for storable commodities. Hausman and Kellogg (2015) implement such a strategy for natural gas using weather data, as weather will cause exogenous shocks to natural gas storage levels – natural gas is used for home heating in the winter, and is used for electricity generation during the summer. Weather exogenously shifts demand for natural gas, and can be used to identify supply parameters. We adopt a similar strategy to identify our drilling equation parameters using deviations from normal population-weighted cooling-degree days (CDDs) and heating-degree days (HDDs).[8] We use a single-month lag and lags of cumulative deviations from degree days over the prior 12-month period. The sums capture the cumulative affect of weather shocks on storage levels (Hausman and Kellogg, 2015). The cooling- and heating- degree day variables are of primary importance as a natural gas demand shifter. Second, we include a hurricane variable, because hurricanes exogenously shift the supply of natural gas and crude oil in the U.S., as gulf coast gas and oil production is often reduced as a result of hurricanes. Third, we use shocks to refinery inputs (thousands of barrels per day) as measured by errors from an autoregression of refinery inputs with three lags, and another autoregression of refinery with three lags and a time trend variable, which represent surprises to crude oil demand associated with refinery maintenance or shutdowns. We use cumulative surprises over the previous 12-month period of both of these variables. Finally, we use lags of the Brent crude oil price to identify global shocks to the crude-oil market, which are unlikely to be affected by drilling in our five estimation basins. All instrumental variables are included in both the first-stage oil price equation and the first stage gas price equation in order to account for the potential impact of demand-side substitution between crude oil and natural gas.

In addition, unobserved drilling shocks are likely to be correlated across the five basins because

[8] Cooling-degree days measure the difference in the average daily temperature from 65 degrees if the temperature is greater than 65 degrees, while cooling-degree days are zero if the temperature is below 65 degrees. Heating-degree days measure the difference in the average daily temperature from 65 degrees if the temperature is below 65 degrees, while heating-degree days are zero if the average daily temperature is above 65 degrees.

they are located in the same geographic region. If capital and labor move between the basins, or if there are regional macroeconomic shocks that affect drilling in all basins, then the errors will be correlated across equations. It is therefore more efficient to estimate equation (38) as a system of equations with oil and gas drilling from all five basins simultaneously. We therefore employ a three-stage least squares approach with 12 equations: five oil drilling equations and five gas drilling equations of the form in equation (38), and two "first stage" equations for the commodity prices that include the instruments described above.

4.1 Data

The data for our estimation of cross-price drilling elasticities were collected from *DrillingInfo*, the *Energy Information Administration (EIA)*, the *National Oceanic and Atmospheric Administration (NOAA)*, and the *National Hurricane Center (NHC)* websites.[9] The data used for our empirical analysis include: real crude oil prices, real natural gas prices, basin-level new crude oil well counts, basin-level new natural gas well counts, deviations from normal population-weighted heating-degree days, deviations from normal population-weighted cooling-degree days, hurricane occurrence and intensity, refinery inputs, and Brent crude oil prices. All variables observed at a monthly frequency.

The price data are spot prices.[10] The crude oil price is the West Texas Intermediate price (WTI) measured in dollars per barrel, which is the U.S. benchmark price for oil, while the natural gas price series is the Henry Hub price (HH) measured in dollars per MMBTU, which is the U.S. benchmark

[9] `www.eia.gov`, `www.noaa.gov`, `www.nhc.noaa.gov`, and `www.bls.gov`.

[10] Estimation was also performed with prompt-month futures and four-month futures for each commodity, and the results were very similar. Results from that analysis are available upon request.

price for natural gas. These price series are inflated to April 2016 U.S. dollars using the BLS Producer Price Index (PPI) for all commodities.

Hurricane data is collected from the National Hurricane Center. This variable is zero if no hurricane made land fall on the U.S. Gulf Coast, and is the maximum Saffir-Simpson Hurricane Wind Scale (commonly known as hurricane category) if one or more hurricanes made land fall on the U.S. Gulf Coast in a particular month.

The *DrillingInfo* dataset provides monthly well-level data which we use in two ways: to construct basin-level dependent variables for the cross-price elasticity equations, and to estimate the decline rates (α_g and α_o) and associated commodity parameters (ψ_g and ψ_o). In order to create the dependent variables, we aggregated the well-level data to construct monthly counts of new natural gas wells and new oil wells in each basin as an approximation of natural gas and crude oil drilling activity. We also constructed monthly total peak production of the target commodity from new wells in each basin as an alternative dependent variable. These monthly counts are available from January 2005 to August 2016, which results in 137 observations per equation in the 3SLS model after accounting for the inclusion of lags.

Summary statistics for the data used in estimation of equation (38) are presented in Table 1. The mean of new wells indicate that our chosen basins include a mix of relatively oil-intensive (Permian) to relatively gas-intensive (East Texas and Forth Worth) basins, although each basin produces both commodities. There is also considerable variation in the explanatory variables and the instruments from which to identify the elasticities. For example, real oil prices vary between roughly \$30 and \$130, while natural gas prices range from less than \$2 to almost \$15 during our sample period.

In order to estimate decline rates, we used monthly well-level production data from *DrillingInfo* to run regressions of the form

$$\ln q_{iwbt} = \alpha_0 + \alpha_{ib} t_w + \eta_w + \nu_{iwbt} \tag{39}$$

where $\ln q_{iwbt}$ is the natural log of production of commodity i from well w in basin b in month t, the variable t_w is a time index for the number of months well w has been in production, and η_w is a well fixed effect. We estimated these equations separately for each basin, so that α_{ib} is an estimate of the average exponential decline rate for wells targeting commodity i in basin b. These estimates are reported in Table 2.

We also used the well-level data to estimate the associated commodity proportions (ψ_g and ψ_o). For each well, *DrillingInfo* reports the commodity that was targeted by the well, the initial peak production of the target commodity, the initial peak production of the associated commodity, and the name of the gas gathering firm if one exists. Although we observe how much of the target and associated commodity were produced from each well during the peak month (typically one or two months following initial production), we are unable to observe how much of the associated commodity that was produced was actually sold. We assume that all associated oil that is produced is sold. Liquids are relatively easy to store and transport by truck even if costly pipeline infrastructure is not present. For associated gas, we assume that if a gas gathering firm is listed on a particular oil well in *DrillingInfo*, then the associated gas from that well is sold. If no gas gathering firm is listed, we assume the gas produced from that oil well is vented or flared. In practice, the amount of associated gas produced may exceed the capacity of the gas gathering infrastructure to remove it and some associated gas will be vented or flared even if gas gathering infrastructure is in place. Our estimates are therefore likely to be an upper bound on the amount of associated gas that is produced and sold. Our purpose in this exercise, however, is not to obtain an exact point estimate of associated commodities that reach the market, but to compare the relative magnitudes of associated commodity production across basins and commodities with the relative signs and magnitudes of cross-price drilling elasticities as articulated in Proposition 2.[11]

The montly well-level associated commodity production data is extremely right-skewed and overdispersed. In order to construct an estimate of ex ante expected associated commodity production for each commodity in each basin, we sum over the peak production of the target commodity and associated commodity from each well type in each basin-month. When summing over associated gas production from oil wells, we only included an observation in the sum if a gas-gathering firm was listed. This procedure produces a monthly basin-level total of peak gas and associated oil production from gas-directed wells, and peak oil and associated gas production from oil-directed wells that is likely to have been sold. We then calculate the ratio of associated to target production for each commodity in each basin-month. We use the median of this ratio for each commodity type, with bootstrapped standard errors, as our estimates of ψ_g and ψ_o in each basin. These estimates are also reported in Table 2, along with the proportion of oil-directed wells in each basin that listed a gas-gathering firm.

[11] With more detailed data on flaring from the Bakken, for example, Fitzgerald and Stiglbauer (2015) find that the quantity of associated gas flared is roughly proportional to the quantity of associated gas produced. Although Bakken production is unique in many respects, this proportional relationship suggests that our procedure will rank the basins appropriately by relative magnitudes of associated gas even if the estimated proportions of associated gas sold may be an upper bound.

5 Results

Table 2 displays the total number of wells in each of our sample basins, the proportion of total wells that were gas or oil directed, the estimated decline rates, the estimated associated-commodity parameters, and the proportion of crude oil wells that are connected to natural gas gathering infrastructure (gg). The last two columns, labeled β_{og} and β_{go} display the basin-level estimated cross-price drilling elasticities. The β_{og} column shows the estimated elasticities of oil drilling with respect to natural gas prices, while the β_{go} column shows the estimated elasticities of natural gas drilling with respect to crude oil price. These are coefficients from the estimation of equation (38) using the natural log of new well counts as the dependent variable. The full set of three-stage least squares drilling equation estimates is reported in Table 3, along with the robustness check using the natural log of peak month production of the target commodity as the dependent variable. The results from the robustness check are largely consistent with our preferred cross-price elasticity estimates reported in Table 2. Table 4 reports the coefficients on the instruments in the price equations from the same three-stage least squares estimation procedure, demonstrating that the Brent oil price, refinery shock, and time trend instruments are significant for both commodity prices while the weather instruments are significant for the Henry Hub price.

Table 2 shows that both cross-price elasticities are statistically significant in the Anadarko Basin, the Fort Worth Basin and the Permian Basin, which are also the largest of the five basins in our sample by total well count. Supply-side links therefore play an important role in producers decisions in these three basins. In the East Texas Basin, by contrast, neither cross-price elasticity is statistically significant and both elasticities are smaller in magnitude than the other basins, although the standard errors of the coefficients are comparable to other basins. This basin also has the smallest associated commodity parameters among the five basins (0.36 for associated gas and 0.025 for associated oil), which limits the importance of the associated commodity channel. East Texas is also the most dominated by a single commodity with 90.6% of wells targeting natural gas, which suggests that input competition from oil producers is not likely to be particularly strong. These are the characteristics we would expect to see in a basin with limited supply-side linkages. On the other hand, we can see from Table 3 that the own-price elasticities in the East Texas Basin are statistically significant and of comparable magnitude to the other basins.

Our estimates of the elasticity of oil drilling with respect to natural gas price are all negative, and four of the five basin-level estimates are statistically significant at at least the 10% level. These negative elasticity estimates are consistent with a cost spillover regime, as laid out in *Proposition 1*, and/or an associated oil regime characterized by *Proposition 2a*. Unfortunately, these negative elasticity estimates do not allow us to concretely distinguish between the cost spillover channel and the associated oil channel. However, it is worth discussing these results in light of the parameters displayed in Table 2. According to *Proposition 2a*, the associated oil channel can cause a negative gas price elasticity of oil drilling if ψ_g or α_o are small, or ψ_o is large. The Chautauqua Platform and Fort Worth Basins have relatively large ψ_g and relatively small ψ_o, suggesting that associated oil production is not likely behind the negative cross-price elasticity. The Anadarko Basin has the largest ψ_o among the basins, although it also has a relatively large ψ_g and α_o. The Permian Basin has the second lowest ψ_g (0.45) and α_o (0.015) and the second largest ψ_o (0.16) suggesting that an associated oil channel could contribute to the negative elasticity.

Four out of our five estimates of the elasticity of natural gas drilling with respect to oil prices are positive, and two of these four positive estimates are statistically significant at at least the 5%

level. The sign of these estimates is consistent with an associated commodity regime characterized by *Proposition 2b.* We find that gas drilling increases by 4.4% when oil prices increase by 10% in the Anadarko Basin, which is consistent with the large associated oil parameter in the Anadarko Basin of 0.17 relative to the other basins. We find that gas drilling increases by 3.1% when oil prices increase by 10% in the Fort Worth Basin (see condition (36)). Although the associated oil parameter is relatively low in the Fort Worth Basin compared to our other sample basins, and the associated gas parameter is relatively large, it is likely very costly to market associated gas from the Fort Worth Basin due to the relatively low proportion of oil wells that are connected to gas gathering infrastructure there and the relatively small share of oil wells in the basin.

The estimate of the elasticity of natural gas drilling with respect to oil price in the Permian Basin is negative and statistically significant in contrast to the other basins used in our estimation. This is likely due to the cost spillover channel rather than the associated gas channel articulated in *Proposition 2a.* First, the estimate of ψ_o is large while ψ_g is small relative to the other basins, which is inconsistent with the associated gas channel in *Proposition 2a.* Second, the Permian Basin has a large number of both oil and natural gas wells, and experienced a drilling boom over our sample period. Therefore, we believe it is likely that input competition is driving supply-side links there.

6 Discussion

Our findings have a variety of implications for energy markets and policy. In this section we describe just a few recent examples.

6.1 Electricity markets and natural gas demand

Most long run electricity supply forecasts envision an increase in both renewable generation and natural gas-fired generation, for a variety of reasons. As intermittent renewable power supply becomes an increasing share of power generation, natural gas-fired generation is a complementary source that can ramp up and down quickly to balance the variable renewable supply. Natural gas-fired power plants can also replace aging coal plants for baseload generation. To the extent that this long-run increase in natural gas demand from the power sector comes to pass and causes an increase in natural gas prices, our results suggest that it may also become more expensive to produce crude oil through the cost-spillover channel. Such a shift may further hasten the adoption of electric cars, which would in turn further increases the demand for natural gas.

6.2 The impact of OPEC

OPEC's 2016 production cut arguably demonstrated that the organization can still influence global oil prices despite the abundance of U.S. shale oil. The WTI price jumped from \$45.29 per barrel to \$49.41 on November 30, 2016, the date of the agreement, and then rose to about \$70 by mid-May 2018. Our results suggest that U.S. natural gas production will increase in response to such moves. Over the same time period, the Baker Hughes North America rotary rig count rose from under 120 to almost 200 active natural gas-directed rigs. The Henry Hub natural gas spot price, after initially jumping by \$0.30 per MMBtu from \$3.02 to \$3.32 on the date of the agreement, fell by about \$0.50 over the same time period, from \$3.32 to about \$2.80 in mid-May 2018. An increase in natural gas production induced by more economical marketing of associated oil has the potential to offset some of the macroeconomic losses that a global oil price shock may have wrought in the pre-shale era, at least in gas-intensive sectors. It also has the potential to shift the location of drilling activity to places where there are particularly “wet” gas wells.

6.3 Carbon budgeting

Carbon budgeting is the process by which a government entity constructs a set of scenarios by which it could meet a particular limit on carbon emissions over a particular window of time. This process is becoming increasingly popular in international climate change negotiations as a way for governments to articulate to each other and to their constituents how they can meet the negotiated targets. Often the scenarios involve reallocating economic activity across different fuel sources, such as a hypothetical shift in sources of electricity generation or transportation fuel requirements. This budgeting process implicitly or explicitly assumes a particular set of tradeoffs between fuel sources that may not capture the supply side links described in this paper. Suppose, for example, that reductions in oil demand from an envisioned shift to more electric cars in a hypothetical carbon budget would, in practice, also reduce natural gas supply through the associated commodity channel. Such an unanticipated reduction in natural gas supply could make it more expensive than expected to generate the additional electricity required for the increase in electric cars.

6.4 Resource taxes and local economic impacts

There is evidence to suggest that there is severance tax competition between state governments (e.g., Maniloff and Manning (2017)). Two states with similar resource abundance want to set severance tax rates that maximize state tax revenue, without driving firms to produce in the competing state instead. However, if state policy makers optimize the tax rates on oil and gas revenues independently, ignoring the spillover of changes in one commodity's tax rate into production of the other commodity, they will not play the optimal tax strategy in the tax competition game. These distortions will likely also spillover into local economies through royalty payments, as suggested by Brown et al. (2016, 2017).

7 Conclusion

A large literature documents the historical statistical relationship, and more recent separation, between crude oil and natural gas prices. This relationship has often been attributed to demand-side substitution, or a "burner-tip parity" relationship in electricity generation and home heating. In this paper, we constructed a model accounting for important supply-side links between crude oil and natural gas markets. We argued that the existence of common inputs such as drilling rigs and specialized labor and materials in the well-completion services market could lead to cost spillovers between oil and gas production, such that increased drilling for crude oil could plausibly increase the cost of drilling natural gas wells and vice versa. Therefore, increased drilling of one type of commodity should be expected to be associated with decreased drilling of the other type of commodity. We also incorporated associated commodity flows into our analysis. Associated commodity flows occur when gas wells produce oil, and/or oil wells produce gas. We showed that if associated commodity flows dominate cost spillovers, then drilling for one commodity may increase in response to a price shock in the other commodity.

Our model characterizes how the cross-price supply responses between oil and gas production depend on the relative scarcity of joint inputs, the rates of associated commodity production, and the price sensitivity of the associated commodity market. Empirical evidence from five major onshore oil and gas producing basins in Texas and Oklahoma confirms the existence of supply side links. The estimated cross-price drilling elasticities indicate that natural gas price shocks reduce onshore oil production rates, but that the effect of oil price shocks on natural gas production rates varies by basin. Specifically, drilling for natural gas increases with oil prices except in the Permian, where oil production is more dominant in general than in the other basins we studied.

There are several caveats to our analysis. Our model does not allow for free disposal of the associated commodity. For example, we do not include the decision to vent or flare associated natural gas from oil wells. This is sufficient for our purposes of characterizing and establishing the marketed supply relationships. Furthermore, previous literature has found that flaring rates are largely proportional to associated gas production rates. Our model could still be adapted to analyze the effect of changes in state venting and flaring policies on drilling and production patterns by making assumptions about changes in the associated commodity flow parameters ψ_i and the associated commodity cost functions $C_\psi(\cdot)$. We also do not include the decision to target wells with a particular rate of associated commodity production. In practice, associated commodity flow rates are mostly fixed at the individual well level, but may vary across potential wells in a way that firms can predict ex ante. The decisions to vent, flare, and/or select wells with heterogeneous associated production could be included at the cost of enormously complicating the model with additional choice variables without altering the basic intuition about supply side relationships. We argue instead that future research on venting, flaring, and/or joint production should take into account the supply-side market links that we characterize in this paper.

As we argue in the discussion section, these links are important for understanding the impacts of a wide array of policies. These include energy policies ranging from state severance taxes to strategic OPEC decisions, as well as any environmental policy that alters the fossil fuel mix, such as ongoing changes in electricity market regulations as well as the increasing use of carbon budgeting. Our findings will take on global importance as more nations discover and extract shale resources at the same time as they negotiate their role in international climate change agreements.

8 Tables and Figures

Figure 1: *Monthly spot prices of crude oil and natural gas. Source: EIA*

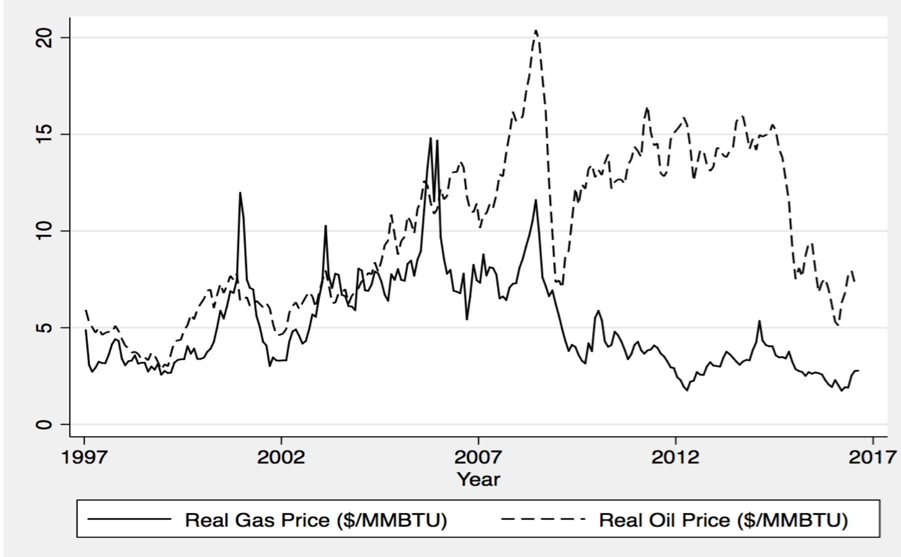

Figure 2: *Monthly crude oil directed rigs and natural gas directed rigs. Source: EIA*

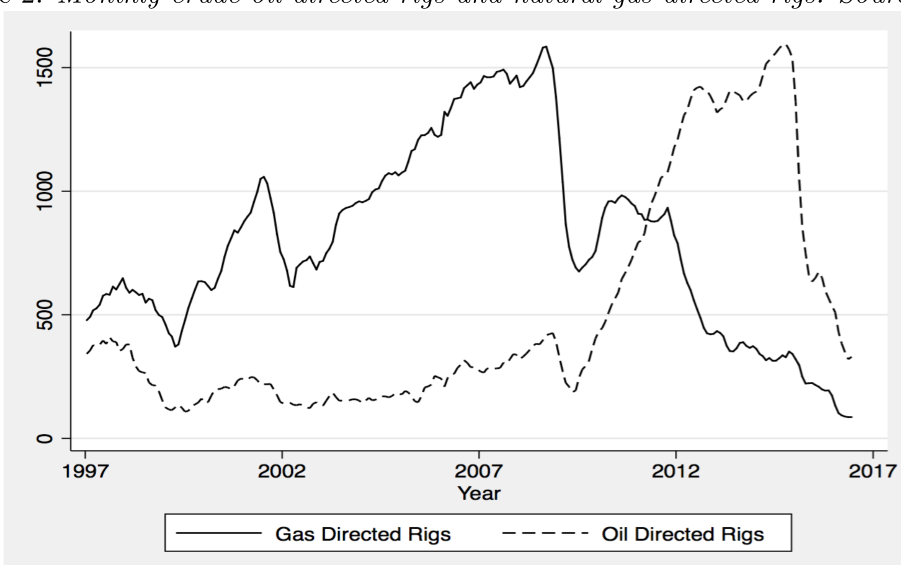

Table 1: Summary statistics.

Variable	Mean	Std Dev	Min	Max
Anadarko Basin New Oil Wells	69.78	48.74	14	205
Anadarko Basin New Gas Wells	103.18	54.00	12	224
Chautauqua Platform New Oil Wells	27.41	17.11	2	120
Chautauqua Platform New Gas Wells	24.21	18.75	1	90
East Texas Basin New Oil Wells	9.59	5.98	2	47
East Texas Basin New Gas Wells	93.51	72.17	6	224
Fort Worth Basin New Oil Wells	23.21	9.52	5	47
Fort Worth Basin New Gas Wells	127.87	76.04	8	292
Permian Basin New Oil Wells	184.72	65.08	89	528
Permian Basin New Gas Wells	68.51	56.68	10	180
WTI Real Spot Price (div 6)	12.45	3.05	5.10	20.40
Henry Hub Real Spot Price	5.11	2.77	1.74	14.80
Brent Oil Price	80.53	26.48	30.70	132.72
Deviation from Normal CDDs	6.20	19.08	-56	63
Deviation from Normal HDDs	-10.99	56.19	-242	153
12-month Cumulative Deviation from Normal CDDs	59.51	89.99	-129	195
12-month Cumulative Deviation from Normal HDDs	-109.47	258.49	-721	305
12-month Cumulative Refinery Input Surprises	328.12	1309.69	-2511.10	3040.26
12-month Cumulative Refinery Input Surprises (w/ time trend)	-324.15	1575.17	-3616.57	2416.63
Hurricane	0.086	0.42	0	3

Table 2: Crude-oil and natural-gas well parameters for each basin.

Basin	Total Wells	% Gas	% Oil	α_g	α_o	ψ_g	ψ_o	gg	β_{og}	β_{go}
Anadarko Basin	24,257	59.6	40.4	0.018	0.022	0.86	0.17	0.76	-0.480***	0.440***
				(0.0001)	(0.0001)	(0.0466)	(0.0157)		(0.1017)	(0.1032)
Chautauqua Platform	7,238	47.0	53.0	0.015	0.013	0.52	0.046	0.63	-0.172*	0.289
				(0.0003)	(0.0003)	(0.0483)	(0.00809)		(0.0950)	(0.2050)
East Texas Basin	14,478	90.6	9.4	0.020	0.018	0.36	0.025	0.63	-0.099	0.142
				(0.0001)	(0.0001)	(0.0494)	(0.00172)		(0.0938)	(0.1047)
Fort Worth Basin	21,225	84.6	15.4	0.017	0.022	0.75	0.042	0.57	-0.127**	0.306**
				(0.0001)	(0.0002)	(0.0400)	(0.00521)		(0.0616)	(0.1263)
Permian Basin	36,002	26.8	73.2	0.014	0.015	0.45	0.16	0.86	-0.283***	-0.251**
				(0.0001)	(0.0002)	(0.00623)	(0.0677)		(0.0479)	(0.1236)

Standard errors in parentheses

Standard errors clustered at well level in well-level fixed-effects estimation of α_i

Bootstrapped standard errors from sample median of ψ_g and ψ_o

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ for β coefficients

Table 3: Crude-oil and natural-gas drilling three-stage least squares results.

	Anadarko Basin		*Chautauqua Platform*		*East Texas Basin*		*Fort Worth Basin*		*Permian Basin*	
	$\mathbf{ln}\,(q_{o,t})$	$\mathbf{ln}\,(q_{g,t})$	$\mathbf{ln}\,(q_{o,t})$	$\mathbf{ln}\,(q_{g,t})$	$\mathbf{ln}\,(q_{o,t})$	$\mathbf{ln}\,(q_{g,t})$	$\mathbf{ln}\,(q_{o,t})$	$\mathbf{ln}\,(q_{g,t})$	$\mathbf{ln}\,(q_{o,t})$	$\mathbf{ln}\,(q_{g,t})$
Dependent Variable: Log of New Well Counts										
ln(Henry Hub)	**-0.480*****	0.362***	**-0.172***	1.296***	**-0.099**	0.545***	**-0.127****	0.181***	**-0.282*****	0.562***
	(0.102)	(0.067)	(0.095)	(0.146)	(0.094)	(0.091)	(0.062)	(0.057)	(0.048)	(0.124)
ln(WTI)	0.720***	**0.440*****	0.684***	**0.289**	0.678***	**0.142**	0.841***	**0.306****	0.568***	**-0.251****
	(0.163)	(0.103)	(0.201)	(0.205)	(0.215)	(0.105)	(0.145)	(0.126)	(0.088)	(0.124)
$\ln(q_{i,t-1})$	0.589***	0.611***	0.473***	0.043	0.161**	0.705***	0.385***	0.850***	0.461***	0.681***
	(0.064)	(0.059)	(0.069)	(0.080)	(0.081)	(0.046)	(0.067)	(0.043)	(0.065)	(0.060)
Constant	0.571*	0.092	0.021	0.101	0.242	0.041	-0.022	-0.359	1.792***	0.999***
	(0.321)	(0.237)	(0.491)	(0.494)	(0.475)	(0.247)	(0.311))	(0.242)	(0.304)	(0.325)
$\mathbf{R^2}$	0.81	0.84	0.34	0.60	0.15	0.92	0.44	0.91	0.70	0.90
Dependent Variable: Log of Peak Month Production										
ln(Henry Hub)	**-0.750*****	0.199***	**-0.339****	1.012***	**-0.859*****	0.382***	**-0.267****	0.295***	**-0.352*****	0.451***
	(0.145)	(0.057)	(0.157)	(0.221)	(0.169)	(0.079)	(0.120)	(0.091)	(0.083)	(0.112)
ln(WTI)	0.581***	**0.516*****	0.715***	**0.689***	1.217***	**0.299****	1.211***	**0.523*****	0.145	**-0.430*****
	(0.174)	(0.112)	(0.268)	(0.383)	(0.307)	(0.130)	(0.248)	(0.185)	(0.105)	(0.154)
$\ln(q_{i,t-1})$	0.645***	0.425***	0.644***	0.005	-0.041	0.545***	0.321***	0.748***	0.752***	0.615***
	(0.056)	(0.068)	(0.062)	(0.082)	(0.079)	(0.062)	(0.077)	(0.054)	(0.048)	(0.064)
Constant	4.074***	7.295***	2.425***	9.437***	8.414***	5.696***	4.143***	2.085***	3.589***	5.845***
	(0.794)	(0.962)	(0.904)	(1.255)	(0.941)	(0.869)	(0.747)	(0.669)	(0.764)	(0.998)
$\mathbf{R^2}$	0.85	0.49	0.54	0.21	0.20	0.66	0.35	0.78	0.85	0.73

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Instrument coefficients in first-stage price equations.

	New Well Counts		Peak Production	
	ln(Henry Hub)	**ln(WTI)**	**ln(Henry Hub)**	**ln(WTI)**
Lag Brent Oil Price	0.0036***	0.0071***	0.0039***	0.0067***
	(0.0008)	(0.0006)	(0.0008)	(0.0007)
12-month Cumulative Refinery Input Surprises	-0.0003***	0.0002***	-0.0003***	0.0003***
	(0.00009)	(0.00007)	(0.00009)	(0.00008)
12-month Cumulative Refinery Input Surprises×Time Trend	0.0003***	-0.0002***	0.0003***	-0.0002***
	(0.00007)	(0.00005)	(0.00007)	(0.00006)
Time Trend	-0.0079***	-0.0058***	-0.0085***	-0.0064***
	(0.0011)	(0.0010)	(0.0012)	(0.0011)
Lag Deviation from Normal CDDs	0.0003	-0.0001	0.0007	0.0000
	(0.0008)	(0.0007)	(0.0009)	(0.0008)
Lag Deviation from Normal HDDs	0.0010***	0.0002	0.0010***	0.0002
	(0.0003)	(0.0002)	(0.0003)	(0.0002)
Lag 12-month Cumulative Deviation from Normal CDDs	-0.0004*	0.0002	-0.0004*	0.0002
	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Lag 12-month Cumulative Deviation from Normal HDDs	0.0001*	0.00005	0.0002***	0.00004
	(0.00007)	(0.00006)	(0.00008)	(0.00006)
Hurricane	-0.0374	0.0194	-0.0244	0.0453
	(0.0397)	(0.0321)	(0.0422)	(0.0361)
Lag Hurricane	0.0396	0.0054	0.0651	-0.0028
	(0.0403)	(0.0325)	(0.0428)	(0.0366)
Constant	2.779***	2.739***	2.830***	2.843***
	(0.682)	(0.129)	(0.168)	(0.144)
$\mathbf{R^2}$	0.86	0.65	0.86	0.66

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

9 Mathematical Appendix

The Hessian matrix associated with identities (24) and (25) is

$$H = \begin{bmatrix} f_{oo} & f_{og} \\ f_{go} & f_{gg} \end{bmatrix}$$

where

$$f_{oo} = \frac{P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} + \frac{\psi_g^2 P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} - C''(q_o+q_g) - C_o''(q_o) - \psi_g^2 C_{\psi_g}''(\psi_g q_o) \leq 0, \tag{40}$$

$$f_{gg} = \frac{P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} + \frac{\psi_o^2 P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} - C''(q_o+q_g) - C_g''(q_g) - \psi_o^2 C_{\psi_o}''(\psi_o q_g) \leq 0 \tag{41}$$

and

$$f_{og} = f_{go} = \frac{\psi_o P_o'\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{\alpha_o(\alpha_o+r)} + \frac{\psi_g P_g'\left(\frac{q_g+\psi_g q_o}{\alpha_g}, s_g\right)}{\alpha_g(\alpha_g+r)} - C''(q_o+q_g) \leq 0, \tag{42}$$

and where $P_i'(z_i, s_i) = \dfrac{\partial P_i'(z_i, s_i)}{\partial z_i}$ for $i = \{o, g\}$. The sufficient conditions for a maximum require that $f_{oo} \leq 0$, $f_{gg} \leq 0$, and $f_{oo}f_{gg} - f_{og}^2 \geq 0$.[12] The conditions on (40), (41) hold as a result of the concavity of consumer surplus and the convexity of the cost functions. For the non-negativity of the determinant of the Hessian we have (dropping dependence from the price and cost functions)

$$\begin{aligned}
&\left[\frac{P_o'}{\alpha_o(\alpha_o+r)} + \frac{\psi_g^2 P_g'}{\alpha_g(\alpha_g+r)} - C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]\left[\frac{P_g'}{\alpha_g(\alpha_g+r)} + \frac{\psi_o^2 P_o'}{\alpha_o(\alpha_o+r)} - C'' - C_g'' - \psi_o^2 C_{\psi_o}''\right] \\
&\qquad - \left[\frac{\psi_o P_o'}{\alpha_o(\alpha_o+r)} + \frac{\psi_g P_g'}{\alpha_g(\alpha_g+r)} - C''\right]^2 \\
&= (1-\psi_o\psi_g)^2 \frac{P_o' P_g'}{\alpha_o\alpha_g(\alpha_o+r)(\alpha_g+r)} - (1-\psi_o)^2 \frac{P_o' C''}{\alpha_o(\alpha_o+r)} - (1-\psi_g)^2 \frac{P_g' C''}{\alpha_g(\alpha_g+r)} \\
&\quad - \psi_o^2\left[\frac{P_o'(C_o'' + C_{\psi_o}'')}{\alpha_o(\alpha_o+r)} - C'' C_{\psi_o}'' - C_o'' C_{\psi_o}''\right] - \psi_g^2\left[\frac{P_g'(C_g'' + C_{\psi_g}'')}{\alpha_g(\alpha_g+r)} - C'' C_{\psi_g}'' - C_g'' C_{\psi_g}''\right] \\
&\quad - \psi_o^2\psi_g^2\left[\frac{P_g' C_{\psi_o}''}{\alpha_g(\alpha_g+r)} \frac{P_o' C_{\psi_g}''}{\alpha_o(\alpha_o+r)} - C_{\psi_o}'' C_{\psi_g}''\right] \geq 0,
\end{aligned}$$

where the last inequality results from the concavity of consumer surplus, the convexity of the cost functions, and the fact that $\psi_o, \psi_g \in [0, 1)$. Thus, the sufficient conditions for maximization are met, and we have that $|H| > 0$.

[12] We have ruled out the possibility that surplus can be increased by inter-temporal reallocation of drilling decisions by assuming that a steady state is reached, thus choosing steady-state drilling rates that maximize surplus at each instant is sufficient for maximizing the value of a drilling program.

Now, we are interested in the comparative statics $\frac{\partial q_g}{\partial s_g}$, $\frac{\partial q_o}{\partial s_o}$, $\frac{\partial q_g}{\partial s_o}$ and $\frac{\partial q_o}{\partial s_g}$, which give the optimal drilling responses to own-commodity and cross-commodity demand shocks. Using the implicit function theorem we have

$$\frac{\partial q_g}{\partial s_g} = \frac{\begin{vmatrix} f_{oo} & -f_{os_g} \\ f_{og} & -f_{gs_g} \end{vmatrix}}{|H|}, \tag{43}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\begin{vmatrix} f_{gg} & -f_{os_o} \\ f_{og} & -f_{gs_o} \end{vmatrix}}{|H|}, \tag{44}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\begin{vmatrix} f_{oo} & -f_{os_o} \\ f_{og} & -f_{gs_o} \end{vmatrix}}{|H|}, \tag{45}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\begin{vmatrix} f_{gg} & -f_{gs_g} \\ f_{og} & -f_{os_g} \end{vmatrix}}{|H|}, \tag{46}$$

where f_{os_o} is the derivative of the oil-drilling identity, (24), with respect to an oil-market demand shock, and f_{gs_o} is the derivative of the gas-market identity, (25), with respect to an oil-market demand shock, and similarly for f_{gs_g} and f_{os_g}. Letting $P_i^s = \frac{\partial P_i(z_i, s_i)}{\partial s_i}$ for $i = \{o, g\}$, we have

$$f_{os_o} = \frac{P_o^s}{\alpha_o + r}, \tag{47}$$

$$f_{gs_o} = \frac{\psi_o P_o^s}{\alpha_o + r}, \tag{48}$$

$$f_{gs_g} = \frac{P_g^s}{\alpha_g + r}, \tag{49}$$

and

$$f_{os_g} = \frac{\psi_g P_g^s}{\alpha_g + r}. \tag{50}$$

Substituting these values into (43), (44), (45) and (46), and simplifying, gives

$$\frac{\partial q_g}{\partial s_g} = \frac{\frac{-P_g^s}{\alpha_g+r}\left[\frac{(1-\psi_g\psi_o)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]}{|H|}, \tag{51}$$

$$\frac{\partial q_o}{\partial s_o} = \frac{\frac{-P_o^s}{\alpha_o+r}\left[\frac{(1-\psi_g\psi_o)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' - C_g'' - \psi_o^2 C_{\psi_o}''\right]}{|H|}, \tag{52}$$

$$\frac{\partial q_g}{\partial s_o} = \frac{\frac{P_o^s}{\alpha_o+r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]}{|H|} \tag{53}$$

and

$$\frac{\partial q_o}{\partial s_g} = \frac{\frac{P_g^s}{\alpha_g+r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]}{|H|}. \tag{54}$$

Given the above comparative statics for optimal drilling responses to demand shocks we can derive the total derivatives of prices with respect to own-commodity and cross-commodity demand shocks, $\dfrac{dP_o(z_o,s_o)}{ds_o}$, $\dfrac{dP_o(z_o,s_g)}{ds_o}$, $\dfrac{dP_g(z_g,s_g)}{ds_g}$, and $\dfrac{dP_g(z_g,s_g)}{ds_g}$. We have

$$\begin{aligned}
\frac{dP_o(z_o,s_o)}{ds_o} = \frac{dP_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{ds_g} &= \frac{\partial P_o}{\partial s_o} + \frac{\partial P_o}{\partial q_o}\frac{\partial q_o}{\partial s_o} + \frac{\partial P_o}{\partial q_g}\frac{\partial q_g}{\partial s_o} \\
&= P_o^s + \left(\frac{P_o'}{\alpha_o}\left\{\frac{-P_o^s}{\alpha_o+r}\left[\frac{(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + C_g'' + \psi_o^2 C_{\psi_o}''\right]\right\}\right. \\
&\quad \left.\frac{\psi_o P_o'}{\alpha_o}\left\{\frac{P_o^s}{\alpha_o+r}\left[\frac{\psi_g(1-\psi_o\psi_g)P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)C'' + \psi_o C_o'' + \psi_o\psi_g^2 C_{\psi_g}''\right]\right\}\right) \Big/ |H| \\
= P_o^s\left\{1 - \frac{P_o'}{\alpha_o(\alpha_o+r)}\left[\frac{(1-\psi_o\psi_g)^2 P_g'}{\alpha_g(\alpha_g+r)} - (1-\psi_o)^2 C'' - C_g'' - \psi_o^2(C_o'' + C_{\psi_o}'' + \psi_g^2 C_{\psi_g}'')\right]\Big/|H|\right\}&
\end{aligned} \tag{55}$$

and

$$\begin{aligned}
\frac{dP_o(z_o,s_o)}{ds_g} = \frac{dP_o\left(\frac{q_o+\psi_o q_g}{\alpha_o}, s_o\right)}{ds_g} &= \frac{\partial P_o}{\partial q_o}\frac{\partial q_o}{\partial s_g} + \frac{\partial P_o}{\partial q_g}\frac{\partial q_g}{\partial s_g} \\
&= \left(\frac{P_o'}{\alpha_o}\left\{\frac{P_g^s}{\alpha_g+r}\left[\frac{\psi_o(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' + \psi_g C_g'' + \psi_g\psi_o^2 C_{\psi_o}''\right]\right\}\right. \\
&\quad \left. - \frac{\psi_o P_o'}{\alpha_o}\left\{\frac{P_g^s}{\alpha_g+r}\left[\frac{(1-\psi_o\psi_g)P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)C'' - C_o'' - \psi_g^2 C_{\psi_g}''\right]\right\}\right) \Big/ |H| \\
&= \frac{P_g^s P_o'}{\alpha_o(\alpha_g+r)}\left[\psi_g C_g'' + \psi_o C_o'' + \psi_g\psi_o^2 C_{\psi_o}'' + \psi_o\psi_g^2 C_{\psi_g}'' - (1-\psi_g)(1-\psi_o)C''\right]\Big/|H|.
\end{aligned} \tag{56}$$

Similarly,

$$\frac{dP_g}{ds_g} = P_g^s\left\{1 - \frac{P_g'}{\alpha_g(\alpha_g+r)}\left[\frac{(1-\psi_o\psi_g)^2 P_o'}{\alpha_o(\alpha_o+r)} - (1-\psi_g)^2 C'' - C_o'' - \psi_g^2(C_g'' + C_{\psi_g}'' + \psi_o^2 C_{\psi_o}'')\right]\Big/|H|\right\} \tag{57}$$

and

$$\frac{dP_g}{ds_o} = \frac{P_o^s P_g'}{\alpha_g(\alpha_o+r)}\left[\psi_o C_o'' + \psi_g C_g'' + \psi_o\psi_g^2 C_{\psi_g}'' + \psi_g\psi_o^2 C_{\psi_o}'' - (1-\psi_g)(1-\psi_o)C''\right]\Big/|H|. \tag{58}$$

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