Division of Economics and Business
Working Paper Series

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Working Paper 2018-02
http://econbus-papers.mines.edu/working-papers/wp201802.pdf

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April 2018

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Working Paper No. **2018-02**
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Comparing applied general equilibrium and econometric estimates of the effect of an environmental policy shock*

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ABSTRACT

We treat the implementation of the carbon tax in British Columbia as a natural experiment and compare the results of econometric estimates of its effects to counterfactual experiments conducted using an applied general equilibrium (CGE) model of the Canadian economy. The comparison allows us to test the theory-driven predictions of the CGE model. It also allows us to test the identifcation strategy of our econometric model, using the CGE model to indicate under what circumstances general equilibrium policy responses might undermine our attempts at statistical inference. Ex post, we find statistically and economically signifcant effects on sectoral employment levels from the carbon tax — with levels falling in the most carbon-intensive sectors and rising in the least carbon-intensive. Ex ante, we predict employment responses of very similar sign and magnitude to our econometric measurements (Pearson correlation coefficient of approximately 0.9). We find no evidence to suggest that our difference-in-difference estimator is likely to be undermined by general equilibrium effects in this policy setting. Finally, we explore the use of the econometric estimates to deepen the empirical content of the CGE model.

JEL **classifications: C68, H23, Q54**
Keywords: climate policy, carbon tax, computable general equilibrium

*Carbone is corresponding author. We acknowledge support from and cooperation with Christoph Böhringer, Tom Rutherford, Randy Wigle, and Environment Canada in developing the computable general equilibrium model. Rivers acknowledges support from the Social Science and Humanities Research Council Canada Research Chairs program. All authors acknowledge support from Carbon Management Canada. The ideas expressed here are those of the authors who remain solely responsible for errors and omissions.

1 Introduction

In this paper, we treat the implementation of the carbon tax in British Columbia (BC) as a natural experiment and compare the results of econometric estimates of its effects to counterfactual experiments conducted using an applied general equilibrium (CGE) model of the Canadian economy. The comparison allows us to test the theory-driven predictions of the CGE model. It also allows us to test the identification strategy of our econometric model, using the CGE model to indicate under what circumstances general equilibrium policy responses might undermine our attempts at statistical inference.

CGE models are frequently consulted to quantify the impacts of prospective environmental policies. For example, the substantial literature on second-best environmental taxation suggests that the design of an economy's pre-existing fiscal system and the interactions of new environmental regulation with it can have substantial effects on the distortionary cost of environment policies (Goulder et al., 1997). This research agenda relies heavily on CGE counterfactual simulations to quantify the general equilibrium responses of factor markets that drive the results. Most of the evidence to date on the impacts of carbon mitigation policies — particularly unilateral policies where important general equilibrium effects of policy arise through trade in energy and energy-intensive goods — comes from CGE analysis (Carbone and Rivers, 2017). Outside of the domain of environmental policy, CGE models are used widely in public finance, international trade and development applications (Shoven and Whalley, 1984).

The main virtue of these models is their theoretical foundation, their ability to capture a full spectrum of channels through which economic theory suggests policy interventions may operate, to generate counterfactual scenarios when policies have no historical analog, and to conduct welfare analysis — the ultimate goal of economic policy analysis in many cases. As tools of empirical analysis, their main shortcoming is their loose connection to empirical evidence based on statistical inference; a number of strong assumptions are required to construct them and key parameter values are typically quantified (or calibrated) using out-of-sample econometric estimates or ad hoc methods. This leads researchers to question the validity of their results.

Testing the validity of these assumptions is challenging however. The settings in which CGE models are most usefully employed — those in which a policy intervention is expected to generate economy-wide changes to prices and sectoral activity levels — also present special challenges for inference, and the dearth of studies with this objective testifies to this fact. When researchers have attempted to validate their models, they have relied on a comparison of the counterfactual changes in outcomes from the CGE model with their analogs in the data measured before and after the economic events of interest - i.e., a simple difference research design (Kehoe, 2005; Valenzuela et al., 2007; Beckman et al., 2011). The concern with this approach is that any change in the economy which coincides with the policy intervention and also affects the outcomes of interest will be interpretted as the causal effect of the policy, a classic form of omitted variable bias. For example, any world event that affects trade flows and was correlated with the implementation of the North American Free Trade Agreement (the subject of Kehoe (2005)) would undermine this research design.

Against this background, the past two decades have seen the rise of experimental and quasi-experimental research designs in the program evaluation literature intended to address exactly the type of omitted variable bias described in the previous example. Quasi-experiments rely on "accidents" of nature or policy to provide plausibly exogenous variation in the policy environment faced by some parts of the economy while leaving other, comparable parts unaffected leading to the difference-in-difference, instrumental-variable and regression-discontinuity designs that now dominate the program evaluation literature (Greenstone and Gayer, 2009).

The BC carbon tax, which we treat as a quasi-experiment in our analysis, plausibly fits this definition. It was introduced in such a way as to make it likely to be exogenous to other events taking place in the BC economy at the time and it was limited to plants located within BC, allowing for the possiblility of comparison of these treated units with plants outside of BC. The first part of our analysis exploits this fact to develop a difference-in-difference estimate of the impacts of the carbon tax on sectoral employment levels. The preferred model uses industry-level outcomes based on: differences in industry-level carbon intensity, differences in the province in which an industry is located and differences across time. The model contains industry×time, and industry×province fixed effects and a differential treatment effect based on the carbon-intensity of different sectors. Therefore, we identify the effects of the carbon tax from industry-province-time variation in the data. We use publicly-available industry-level economic and carbon emission data from Statistics Canada. We then compare these estimates to the ex ante predictions of a province-level CGE model of the Canadian economy. Subject to the maintained assumptions of the econometric estimator, this allows us to validate the results of the CGE model using a techique that has the potential to purge the most obvious sources of omitted variable bias present in past validation exercises.[1]

Using the econometric model, we find statistically significant impacts of the carbon tax across sectors that differ in benchmark carbon intensity. The model predicts a pattern in which employment in the most carbon-intensive sectors (e.g., coal production and cement, for example) declines while employment in the least carbon-intensive sectors (e.g., services) increases relative to the average sector. The most affected sectors decline by 10-15%, while growth sectors expand on the order of 0-5%.

The CGE model produces a similar pattern of changes across sectors. The sign of the predictions of changes in employment caused by the carbon tax - as estimated by the CGE and econometric approaches - agree across the preferred specifications of the models 81-98% of the time (depending on weighting scheme adopted). The Pearson correlation coefficient, a measure of linear association that captures agreement in both sign and relative magnitudes of sectoral changes, range from 0.86-0.95. Overall, the CGE model predicts sectoral employment responses to the carbon tax that are very close in magnitude to those predicted by the econometric results. We take this as encouraging evidence that a 'typical' CGE model can usefully predict the economic response to an environmental policy shock.

There are, however, important reasons to question the validity of quasi-experimental research designs like the one presented here in the evaluation of policies with economy-wide impacts. Their identification relies on finding treatment and control units that isolate the effects of interest. In our application, for example, we must assume that plants located outside BC's borders are unaffected by the carbon tax (the so-called stable unit treatment value assumption (SUTVA)). Yet these plants

[1] McKitrick (1998) presents a related critique, comparing versions of CGE models in which the key parameters are estimated econometrically, based on CES functions or alternative flexible functional forms.

may be competitors with BC firms in both input and output markets. A loss of competitiveness of carbon-intensive industries in BC due to the new tax could cause output and exports of competitors in the rest of Canada to rise in order to capture displaced demand for BC-produced goods. Similarly, lower demand for BC-based goods may lower wages for workers leading them to seek employment in other provinces. As we have noted, a large literature on the effects unilateral climate policy — based primarily on CGE counterfactual analysis — emphasizes the importance of trade in energy, energy-intensive goods and basic factors as a source of offsetting changes in economic activity in unregulated regions of the world economy, so called "carbon leakage" effects, which may undermine the effectiveness of these designs (Carbone and Rivers, 2017).

This concern leads to the second part of our analysis, which exploits the fact that CGE models are designed to capture theory's prediction regarding the magnitude and sign of these types of economy-wide interactions. We use it to explore the importance of these influences on the impacts of the BC carbon tax. We then use these predictions to test validity of our econometric research design. Subject to the maintained assumptions of the theory underlying the CGE model, we can determine the degree of bias introduced by economy-wide influences — an issue on which quasi-experimental econometric analyses are typically silent.[2]

By using "pseudo-data" generated from the CGE model, we estimate our econometric model. We find that the difference-in-difference estimator predicts a very similar pattern of impacts when the pseudo-data either contains or is purged of general equilibrium effects that could contaminate our control units. This indicates that the SUTVA assumption is expected to hold and thus general equilibrium responses are unlikely to undermine our econometric research design.

Because of the close correspondence between the effects generated by our two models and because we find little potential for contamination using the diff-in-diff design, we proceed in the final part of the analysis to use the econometric model to calibrate some of the key parameters that sensitivity analysis shows to have an important influence on the results of the CGE counterfactual analysis. Taking the CGE model directly to data would be computationally complex. The diff-in-diff model serves as a simplified description of the key behaviors which allows us to base some of

[2]See Chetty (2009), Heckman (2010), Keane (2010) and Kuminoff and Pope (2014) for related discussions of the use of structural models to evaluate treatment effects in program evaluation.

the parameter values of our CGE model on statistical inference, a method referred to as indirect inference (Gourieroux et al., 1993). This approach would allow the analyst to conduct welfare analysis of the BC carbon tax policy — something that is not possible using the typical program evaluation framework — using a CGE model based on a stronger empirical foundation than the typical, calibrated model. It could also be used to conduct prospective analyses of future changes to Canadian carbon policy.

To our knowledge, ours is the first attempt to compare the performance of a CGE model and a quasi-experimental econometric model. It is also the first attempt to evaluate the performance of a CGE model designed to study the effects of a carbon tax or any other environmental regulation.

The BC carbon tax has a number of features that make it well suited to our purpose. First, it was applied uniformly on all combustion-related greenhouse gas emissions in the province, which makes it straightforward to simulate in a CGE framework. In contrast, most other climate and environmental policies specify particular technological parameters, which is much more challenging to simulate in a CGE framework. Second, it reached \$30 per tonne CO_2 in 2012 and was first implemented in 2008. Thus, the price signal is likely to be sufficiently strong and the history of the policy sufficiently long that its impacts may be measured in the data. Third, the economy had very little time to anticipate the policy. Only a year passed in between the time the BC government first mentioned that it was contemplating a carbon tax and when it actually came into force. Only five months passed in between the time the government made the official announcement about the structure of the policy and the time it was implemented. As a result, it is likely that comparing behavior immediately before and after implementation captures the outcomes of interest. Fourth, the BC carbon tax is revenue neutral. That is, all of the carbon tax revenue is returned to individuals and businesses through reductions of other taxes. The corporate income tax rate and the two lowest personal income tax rates were reduced by 5 percent.[3] The BC carbon tax presents a unique opportunity to evaluate a specific set of general equilibrium responses generated by CGE models that are central to the theory of second-best environmental taxation. Fifth, British Columbia intro-

[3] In addition, to protect low-income households, the government gives them a lump-sum credit. In 2012-2013, \$1.4 billion was credited back to individuals and businesses. In fact, tax credits exceeded the tax revenue by \$260 million. This excess is estimated to decline to only \$20 million in 2013-2014.

duced the carbon tax unilaterally. The specter of a loss of competitiveness in pollution-intensive, trade-exposed domestic industries and of carbon leakage (offsetting increases in emissions outside of the regulated jurisdiction) has played an important role in preventing many countries from adopting emission controls. CGE models with descriptions of the system of interregional trade form the principal piece of evidence on the magnitude of the competitiveness and leakage effects (Carbone and Rivers, 2017). Thus, our experiments have the potential to shed light on this set of issues as well.

We focus on comparing sectoral changes in employment across our two models. There are a number of reasons to choose this indicator. First, examining the impact of a carbon tax on sectoral activity is of interest to policy makers, since this is closely related to the concepts of competitiveness and emissions leakage, both of which are important considerations for the development of cost-effective and politically acceptable carbon policy (Carbone and Rivers, 2017). Second, impacts on employment are themselves central to the political acceptability of climate policy and there is only limited empirical evidence on the effect of a carbon tax on employment (Hafstead and Williams III, 2016; Yamazaki, 2017). Third, employment is relatively straightforward to measure without error, so it is a useful indicator of the sector-level impact of the carbon tax.

The rest of the paper proceeds as follows. In the following section, we describe both the econometric and computable general equilibrium models that are used, in addition to the sources of data. In section 3, we present results from each model describing the economic response to the introduction of a carbon tax, and also formally compare the results of the two models. In section 4, we focus on uncertainties in the two models, and explore whether it is possible to use the models jointly to narrow the uncertainties. Finally, section 5 concludes.

2 Models

2.1 Econometric model

We econometrically estimate changes in employment in response to the introduction of the carbon tax in British Columbia using a difference-in-difference framework combined with a treatment intensity variable. We compare differences in these variables (i) before and after the introduction of the carbon tax, (ii) in British Columbia and other provinces, and (iii) in sectors with high and low carbon intensity. Given the assumptions described in the prior section, the difference-in-differences framework allows us to isolate the causal effect of the carbon tax on sectoral performance in British Columbia. Our approach builds on prior work by Yamazaki (2017) and Rivers and Schaufele (2014).

Formally, our approach for estimating the impact of the carbon tax on employment is to estimate:

$$\ln L_{ijrt} = \beta_1(EI_{ijr} \times \tau_{rt}) + \beta_2\tau_{rt} + \lambda^1_{ijr} + \lambda^2_{ijt} + \epsilon_{ijrt} \tag{1}$$

where L_{ijrt} is employment of industry i in sector j in region/province r at time t, EI_{ijr} is the emissions intensity of industry i in sector j in region r, measured in tonnes of greenhouse gases per dollar of output,[4] τ_{rt} is the value of the carbon tax in province r in year t, λ are fixed effects, and ϵ_{ijrt} is a normally distributed error term. Successful identification of the β parameters of interest is contingent on the fixed effects absorbing potentially confounding variables. λ^1_{ijr} is an industry-sector-province fixed effect that absorbs the average employment by industry in each province. λ^2_{ijt} is a industry-sector-time fixed effect that absorbs any common shocks by industry sector, for example as a result of changes in commodity prices or national policy. Threats to identification of β_2 come in the form of disturbances that are correlated with τ_{rt} and not absorbed by the fixed effects. For example, if British Columbia implemented other province-wide policies concurrently with the carbon tax, or if there were shocks to labour supply in British Columbia that varied over time, estimates of β_2 would be biased. Threats to identification of β_1 come in the form of disturbances that are correlated with $EI_{ijr} \times \tau_{rt}$. For example, changes in the domestic demand in BC for a particular commodities over time that are correlated with the carbon tax could bias estimates of β_1.

[4]We explain the measurement of emissions intensity as well as the industry-sector concordance in a later section.

2.2 Computable general equilibrium model

We use a static multi-sector, multi-region computable general equilibrium (CGE) model of the Canadian economy to simulate changes in sectoral employment in response to the introduction of the carbon tax in British Columbia. The model is a "standard" implementation of an energy-focused computable general equilibrium model. The model has previously been used in several other applications for assessment of climate change policy in Canada.[5] This section includes a non-technical overview of the model. A more formal model description is provided in the Appendix to this article.

The model captures characteristics of provincial (regional) production and consumption patterns through detailed input-output tables and links provinces via bilateral trade flows. Each province is explicitly represented as a region, except Prince Edward Island and the Territories, which are combined into one region. The representation of the rest of the world is reduced to import and export flows to Canadian provinces which are assumed to be price takers in international markets. To accommodate analysis of energy and climate policies the model incorporates rich detail in energy use and greenhouse gas emissions related to the combustion of fossil fuels.

The model features a representative agent in each province that receives income from three primary factors: labour, capital, and fossil-fuel resources.[6] There are three fossil resources specific to respective sectors, namely, coal, crude oil and gas. Fossil-fuel resources are specific to fossil fuel production sectors in each province. Labour is treated as perfectly mobile between sectors within a region, but not mobile between regions. A portion of the capital stock is treated as mobile between sectors and provinces, while a portion is treated as fixed. The model incorporates details of direct and indirect taxes which are received by the provincial or federal governments in order to finance public services.

The choice of sectors in the model has been to keep the most carbon-intensive sectors in the available data as separate as possible. The energy goods identified in the model include coal, gas,

[5]For example, see Böhringer et al. (2015) for an application to burden-sharing, Böhringer et al. (2016) for an application to fiscal federalism, and Beck et al. (2015) for an application related to the incidence of carbon taxes.

[6]Labour supply is endogenous, following the specification in Ballard (2000). Uncompensated (compensated) elasticity of labour supply is 0.05 (0.3). Land use associated with agricultural production and forestry is therefore not explicitly accounted for, but instead treated as part of the specific capital stock of the relevant sector.

crude oil, refined oil products and electricity. This disaggregation is essential in order to distinguish energy goods by carbon intensity and the degree of substitutability. In addition the model features major energy-intensive industries which are potentially those most affected by emission reduction policies.

We describe in some detail the structure of the model with a focus on sectoral production and trade in the sub-sections below. We focus on these aspects of the model since they are the key model structures that affect the response to carbon policy, which we compare with the econometric evidence. Further details on other aspects of the model are available at Böhringer et al. (2015).

Production

Production of commodities in each region and sector pair (Y_{jr}) is captured by multi-level constant elasticity of substitution (CES) cost functions describing the price-dependent use of capital, labour, energy and materials. Production is assumed to occur by perfectly competitive constant-returns-to-scale firms. At the top level of the production function, a composite of non-energy intermediate material demands (M_{jr}) is combined with an aggregate of energy, capital, and labour (KLE_{jr}) subject to a constant elasticity of substitution (σ^M):[7]

$$Y_{jr} = \left(\theta^M_{jr} (M_{jr})^{\frac{\sigma^M - 1}{\sigma^M}} + \theta^{KLE}_{jr} (KLE_{jr})^{\frac{\sigma^M - 1}{\sigma^M}} \right)^{\frac{\sigma^M}{\sigma^M - 1}} \tag{2}$$

where θ^M_{jr} is the value share of intermediate inputs (M) in the production of Y. θ is similarly defined in the equations below.

The intermediate good composite is a fixed (Leontief) composite of the M individual intermediate inputs, each of which is an Armington composite of imports and domestic production (as described below):

[7] We drop subscripts on σ^M and other elasticities of substitution in the prodution function to reduce clutter, although these are differentiated by sector as described below.

$$M_{jr} = \min(\theta_{jr}^{M_1} A_{1r}^{j}, \ldots, \theta_{jr}^{M_M} A_{Mr}^{j}) \tag{3}$$

where A_{kr}^{j} is Armington good k in region r used as an input to sector j $(k = j)$. Other inputs include energy, capital, and labour. In the main model specification, capital and labour are aggregated into a CES composite (V_{jr}), which is then combined with an energy composite (E_{jr}):

$$KLE_{jr} = \left(\theta_{jr}^{E} (E_{jr})^{\frac{\sigma^{E}-1}{\sigma^{E}}} + \theta_{jr}^{V} (V_{jr})^{\frac{\sigma^{E}-1}{\sigma^{E}}} \right)^{\frac{\sigma^{E}}{\sigma^{E}-1}} \tag{4}$$

where

$$V_{jr} = \left(\theta_{jr}^{K} (K_{jr})^{\frac{\sigma^{L}-1}{\sigma^{L}}} + \theta_{jr}^{L} (L_{jr})^{\frac{\sigma^{L}-1}{\sigma^{L}}} \right)^{\frac{\sigma^{L}}{\sigma^{L}-1}} \tag{5}$$

Values for the elasticities of substitution between capital and labour, as well as between value added and energy differ by sector and are drawn from the econometric work of Okagawa and Ban (2008).[8]

The energy composite is a nested CES aggregate of electricity, gas, refined petroleum products (oil), and coal. Specifically, the aggregate energy input is defined as a CES function of electricity and the composite of coal, oil and gas; and the composite, coal, oil and gas is a CES function of coal and a CES aggregate of oil and gas:

$$E_{jr} = \left(\theta_{jr}^{ele} (ele_{jr})^{\frac{\sigma^{ELE}-1}{\sigma^{ELE}}} + (1 - \theta_{jr}^{ele}) (CGO_{jr})^{\frac{\sigma^{ELE}-1}{\sigma^{ELE}}} \right)^{\frac{\sigma^{ELE}}{\sigma^{ELE}-1}} \tag{6}$$

[8]In the simulations below, we test the impact of an alternative nesting structure in which labour is combined with a CES aggregate of capital and energy, also based on econometric estimates from Okagawa and Ban (2008). We also test the impact of replacing the econometrically-estimated elasticities from Okagawa and Ban (2008) with those from Dissou et al. (2012). This results in a total of four alternative production function specifications (two different nesting structures and two different sets of econometric estimates).

where:

$$CGO_{jr} = \left(\theta_{jr}^{col}(col_{jr})^{\frac{\sigma^{COA}-1}{\sigma^{COA}}} + (1-\theta_{jr}^{col})(GO_{jr})^{\frac{\sigma^{COA}-1}{\sigma^{COA}}}\right)^{\frac{\sigma^{COA}}{\sigma^{COA}-1}} \tag{7}$$

and

$$GO_{jr} = \left(\theta_{jr}^{gas}(gas_{jr})^{\frac{\sigma^{OIL}-1}{\sigma^{OIL}}} + (1-\theta_{jr}^{gas})(oil_{jr})^{\frac{\sigma^{GO}-1}{\sigma^{OIL}}}\right)^{\frac{\sigma^{OIL}}{\sigma^{OIL}-1}}. \tag{8}$$

Elasticities of substitution for energy goods are $\sigma^{ELE} = 0.25$, $\sigma^{COA} = 0.5$, and $\sigma^{OIL} = 0.75$. These elasticities take on similar values in other CGE models (see e.g., Paltsev et al. (2005)). They are also consistent with substantial empirical evidence available in this regard (e.g., (Stern, 2012)).

In the production of fossil fuels (coal, crude oil and natural gas), the production function is similar to that described above, except the capital-labour-energy-materials aggregate is combined with a fossil fuel specific resource at the top level. The elasticity of substitution between this sector-specific resource and the other inputs is calibrated to reflect empirical evidence on fossil fuel supply elasticities as described in Rutherford (2002).

In all of the simulations we consider, we take technology as exogenous. That is, firms can move along isoquants in response to changes in (relative) prices, but isoquants are fixed. This assumption effectively rules out innovation as a response to changes in emission prices.

Trade

Bilateral trade between provinces as well as between each province and the rest of the world is specified following the Armington (1969) approach, which distinguishes domestic and foreign goods by origin. Output produced in each sector is supplied to each of the domestic regions and the rest of the world. Given the ratio of regional and external prices a constant elasticity of transformation (CET) function determines quantities supplied to the domestic and export markets:

$$Y_{jr} = \left(\theta^D_{jr}(D_{jr})^{\frac{\eta-1}{\eta}} + (1-\theta^D_{jr})(X_{jr})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$

where D_{jr} is production of good j in region r that is sold on the domestic market and X_{jr} is the volume sold to export markets (X_{jr} is a composite of exports to other provinces s (X^s_{jr}) and to other countries (X^F_{jr})). η is the elasticity of transformation between domestic goods and exports. In a similar way, all intermediate and final demand goods correspond to a CES composite that combines the domestically produced good and the imported goods from other provinces and the rest of the world.

$$A_{jr} = \left(\theta^A_{jr}(H_{jr})^{\frac{\sigma^{DM}-1}{\sigma^{DM}}} + (1-\theta^A_{jr})(M_{jr})^{\frac{\sigma^{DM}-1}{\sigma^{DM}}}\right)^{\frac{\sigma^{DM}}{\sigma^{DM}-1}},$$

where H_{jr} is good j produced domestically (within Canada) that is consumed in region r, and M_{jr} is imports of good j to region r from the rest of the world. The commodity H_{jr} is itself an aggregate of goods produced in each region of Canada:

$$H_{jr} = \left(\theta^{H_i}_{jr}(D_{jr})^{\frac{\sigma^{PP}-1}{\sigma^{PP}}} + \sum_{s\neq r}\theta^{H_s}_{jr}(X^r_{is})^{\frac{\sigma^{PP}-1}{\sigma^{PP}}}\right)^{\frac{\sigma^{PP}}{\sigma^{PP}-1}}.$$

All Canadian provinces are assumed to be price takers in the world market. There is an imposed balance of payment constraint between Canada and the rest of world aggregate. To implement this constrain, we fix the current account surplus exogenously at the benchmark level.

The elasticities η, σ^{DM}, and σ^{PP} determine the changes in exports and imports as a function of changing relative prices of domestic and foreign goods. In the benchmark model calibration, we set $\eta = \sigma^{DM} = 4$ and set $\sigma^{PP} = 2\sigma^{DM}$. We do now know of a source that empirically estimates Armington elasticities for Canadian provinces, and so these parameters are chosen to roughly reflect empirical literature from other regions (Donnelly et al., 2004). We set σ^{PP} at twice the level of σ^{DM} to reflect the 'border effect' which suggests that goods move more easily over provincial and state borders than over national borders (Anderson and Van Wincoop, 2003). A similar approach is taken in Caron et al. (2014).

Simulation

Using the CGE model, we conduct an analysis of the impact of the introduction of the carbon tax in British Columbia. The tax was phased in over five years starting in 2008, and reached a value of \$30/t CO_2 in 2012. This tax value was applied on all combustion greenhouse gas emissions in the province, while non-combustion emissions were not covered by the tax (in total, the tax covered about 75% of total emissions). We ignore transitional dynamics associated with the tax's introduction, and focus on its 2012 level (BC's government has stated that the tax will be held at this level until at least 2018). To analyze the effect of the tax, we construct two model simulations: one in which the tax is applied to all fossil fuel combustion in British Columbia, and one in which no tax is applied. We infer the effect of the tax in the CGE model by comparing these two counterfactual simulations.

British Columbia's carbon tax was introduced as part of a broader *environmental fiscal reform*, in which revenues from the tax were used in part for lump-sum transfers to low-income households, and in part for reducing rates of pre-existing personal and corporate income taxes in the province. We replicate the revenue recycling scheme that was implemented by the province in our analysis, as described in detail in Beck et al. (2015). One feature of the implementation of the tax is that government carbon tax revenues were slightly lower than associated tax rebates, such that the deficit position of the provincial government increased slightly in conjunction with the tax. We replicate this change in our simulation of the tax.

2.3 Data sources and reconciliation

In order to make meaningful comparisons between the two modeling approaches, we need to ensure that data used for both models is comparable. This section explains the sources of data used for each model, and the steps taken to ensure comparability.

The CGE model adopts the 'calibration' approach to parameterization, in which cost and income share parameters are drawn from the benchmark social accounting matrix, and free elasticity parameters are drawn from estimates in the published literature (as described above). Statistics Canada input-output and final demand data provides the foundation for the social accounts matrix (Statistics Canada, 2006a,b). This data is not available at a high sectoral resolution at the provincial level, and so we make use of disaggregated national data as well as proprietary data from Environment Canada to further disaggregate certain energy-intensive sectors, as well as to obtain data on sectoral greenhouse gas emissions. We use economic data from 2007 (prior to introduction of the carbon tax) as the benchmark data source.

Table 1 summarizes key features of the resulting data set underlying the CGE model. There are 17 sectors in each region of the model, including three primary energy sectors (natural gas, crude oil, and coal), two secondary energy sectors (refined petroleum products and electricity), five manufacturing sectors (pulp and paper, primary metals, chemicals, cement, and other), as well as several other sectors that are generally less energy- and emissions-intensive.[9] The largest sectors in the economy produce few emissions, notably the service, government, construction, other manufacturing, and trade sectors. The sectors that produce the largest amount of emissions per unit output are the coal mining and cement, followed by the crude oil production, primary metal manufacturing, and transport sectors. Manufactured goods and energy goods are highly traded. Figure 1 visually summarizes key sectoral data, and shows the sectors potentially most 'exposed' to carbon prices: those with high emissions intensity as well as a large export share (which likely limits ability to pass through the carbon cost to final consumers). These sectors include coal mining, cement, and the crude oil extraction sectors, and to a lesser extent the heavy manufacturing sectors (chemicals, pulp and paper), the transport sector, and the natural gas production sector. Each of these represents a small share of the total BC economy.

[9]Note that the emissions intensity reported in Table 1 includes both the direct emissions as well as the indirect emissions that originate in the province of British Columbia. We calculate indirect emissions based on the benchmark input-output matrix, using the method described in Rutherford (2010). We only include emissions covered by the BC carbon tax in this calculation (i.e., due to fossil fuel combustion in BC), since this is the basis for estimating the impact of the tax.

The econometric approach is based on data on sectoral employment collected by Statistics Canada's Survey on Employment, Payrolls, and Hours (SEPH). We use annual data by province and NAICS sector[10] The Statistics Canada data reports the total number of employees in each sector, province, and year. While the CGE model uses data on 17 sectors (indexed by j in the discussion above) per region, the econometric model uses data on about 80 industries (indexed by i). We map these disaggregate sectors to the more aggregate CGE sectors, as shown in Appendix Table 6.

Our econometric analysis also employs data on the emissions intensity (and trade intensity) of each industry sector. Unfortunately, Statistics Canada does not maintain data on the emissions intensity of industrial sectors disaggregated at a provincial level. In a similar analysis, Yamazaki (2017) uses the national emissions intensity as a proxy for the emissions intensity for each sector and region. We build on that approach here, but also make use of the provincial sector-level emissions intensity from the CGE model database. This is important, since in some cases there are significannt differences between national emissions intensity in a given sector and the corresponding provincial measure (e.g., in electricity generation, where some provinces produce virtually no emissions and others produce significant quantities per unit of electricity generated). In particular, we construct the emissions intensity for sector i in region j in province r based on national Statistics Canada industry level data ($\tilde{EI}_i$) and provincial-sector emissions intensity from the National Accounts ($\bar{EI}_{jr}$) as follows:

$$EI_{ijr} = \tilde{EI}_i \times \frac{GDP_{jr} \times \bar{EI}_{jr}}{\sum_k GDP_{kjr} \times \tilde{EI}_i}$$

This measure preserves the intra-sectoral variation in emissions intensity that is observed in the national data, but ensures that aggregate sector emissions intensity matches the provincial data. Using this measure ensures that the econometric results are directly comparable to the results generated by the CGE model.

[10]CANSIM series 281-0024.

3 Results

3.1 Econometric model results

Table 2 shows the results corresponding to the estimation of (1). The coefficient on tax ($\hat{\beta}_2$ in (1)) is based on a difference-in-difference estimation that compares employment in a particular industry in British Columbia after the carbon tax was implemented with employment in the same industry and province prior to the implementation of the tax. Any shocks common to particular industry sectors that transcend provinces are absorbed by industry-time fixed effects. The coefficient on tax$\times$emissions intensity ($\hat{\beta}_1$ in (1)) is estimated using a difference-in-difference framework combined with a treatment intensity variable - the emissions intensity of different industry sectors - as an additional control. Different columns in Table 2 correspond to the inclusion of different fixed effects in the estimation of (1).

In the first column of the table, we include fixed effects for time (year), industry, and province. Results suggest that sectors with higher carbon intensity are more adversely affected by the imposition of the carbon tax. In subsequent columns of the table, we successively include a more complete set of fixed effects to control for unobserved time-varying shocks or cross-sectional characteristics that could bias the results. Our preferred specification, which is described above as equation (1), is given in column (4), and includes industry-province fixed effects and industry-time fixed effects. These fixed effects respectively control for unobserved heterogeneity and events that vary systematically by industry over time (for example, exchange rates, macro-economic policy, changes in trade policy, changes in technology, or changes in commodity prices), as well as controlling for unobserved factors that cause some industries to be larger in some provinces than others (for example, different resource bases, policy, or preferences). Identification of the effects in the model is contingent on there being no other unobserved heterogeneity that is correlated with the treatment variables in the model.

In column (4), which we focus on, the model suggests that sectors with higher emissions intensity are more likely to experience reductions in employment as a result of the carbon tax than sectors with low emissions intensity. The coefficient on Tax $\times$ intensity suggests that for a sector with emission intensity of 1 tonne CO_2 per \$1,000, a \$1 increase in the carbon tax is associated with a 0.53 percent reduction in employment compared to a sector with no emissions.[11] The coefficient is estimated precisely, and is similar in magnitude to Yamazaki (2017) who uses a similar approach and data. The coefficient on Tax suggests that for a sector with no emissions, employment increases by 0.2 percent when the carbon tax increases by \$1. This coefficient is not statistically significant and standard levels. This is perhaps unsurprising, as the overall effect of the carbon tax on employment is likely to be small, and the number of observations available here is also small. These two coefficients can be used to generate predicted values for the impact of the carbon tax on employment. For example, the Primary Metal Manufacturing sector (which has an emissions intensity of 0.619 tCO_2/\$1,000) would be predicted to reduce employment by about 4.3% as a result of the carbon tax at \$30/t CO_2.[12]

Predicted values for each sector are shown as point estimates in Figure 2. The econometric model predicts a reduction in output and employment in the emissions-intensive sectors of the economy, and a growth in output and employment in the less emissions-intensive sectors. Importantly, the largest sectors of the economy - services, wholesale and retail trade, other manufacturing, government, and construction - all have low emissions intensity. Conversely, the emission-intensive sectors such as the cement, coal, and pulp and paper sectors represent a much smaller fraction of the overall economy.

3.2 Computable general equilibrium model results

Figure 3 shows the key results from the CGE model pertaining to sectoral employment, output, and exports. The sectors predicted by the model to experience the most significant impacts in terms of employment losses are the coal (COL) and cement (CEM) sectors, which the model suggests reduce employment by 17% and 12%, respectively. In addition, the model suggests employment losses of 7 to 10% in the oil refining (OIL), primary metal manufacturing (PRM), and transport (TRN) sectors. As shown in Table 1, these sectors have high emission intensity relative to other sectors in

[11]The emissions intensity measure is described above. Briefly, we sum direct and indirect emissions covered by the carbon tax originating in BC and divide by gross output.

[12]We calculate predicted values using $\Delta \hat{L}_i = \exp(30 \times (\hat{\beta}_1 \times EI_i + \hat{\beta}_2)) - 1$.

BC. Most of these sectors are also highly traded. These two factors result in these sectors being particularly sensitive to the tax. In contrast, the model suggests increases in employment in the agriculture (AGR), construction (CON), manufacturing (MFR), services (SER), and trade (TRD) sectors, all of which are low greenhouse gas intensity sectors.

3.3 Comparison of results

In this section, we compare results from the econometric models and from the computable general equilibrium model. We focus here on comparing point estimates, although the prior analysis suggests that both the econometric and CGE analysis are associated with uncertainty.

Figure 4 graphically compares point estimates of sector changes in employment as a result of a \$30/t CO_2 tax in British Columbia from the econometric model and from the computable general equilibrium model. In the figure, the dashed black line has an intercept of zero and a slope of 1, such that all points would fall along this line if the two models were identical. There appears to be relatively good visual concordance of the results. Sectors for which the CGE model predicts a large negative impact - in particular the coal mining and cement sectors - are also predicted to experience a large negative impact based on the econometric results. Sectors for which the CGE model predicts will experience a benefit as a results of the carbon tax - in particular the low emission sectors such as services, wholesale and retail trade, and other manufacturing - are also predicted to experience a benefit based on the econometric results.

Table 4 is a formal comparison of the results. In this table, we calculate the correspondence of the predictions from the econometric and CGE models according to three metrics. First, we evaluate the concordance in signs. If the two models predict the same sign for a particular output, we assign a value of 1, and if the two models predict opposite signs (e.g., negative and positive), we assign a value of 0. In Figure 4, this involves determining the number of observations in quadrants 1 and 3 (with equal signs) compared to the number of observations in quadrants 2 and 4 (with opposite signs). The results of this calculation are given in the first row of Table 4. In the first column, we report unweighted results. These suggest that the CGE model and econometric model were in agreement about the sign of impacts by sector for 81 percent of the sectors in the model.

As can be seen in Figure 4, for three sectors – oil refining, electricity generation, and pulp and paper manufacturing) – the econometric model predicts an increase in employment, while the CGE model predicts and reduction in employment. In other cases, predictions from the two models are identical in sign. Moreover, even for the three sectors where the models were not in agreement, the sector was only just outside of the boundaries of the 1st or 3rd quadrant. The second column of Table 4 shows the same calculation, but this time weighted according to the benchmark output of each sector. In this case, agreement between the two models is nearly complete: the two models predict the same directionality of change in activity for nearly the full amount of economic activity.

The second row of Table 4 reports the correlation between predictions arising from the two modeling approaches. Formally, we compare $\Delta\hat{L}_i$ in each model, where a superscript ^{E} refers to the econometric model and a superscript ^{S} refers to the simulation model, and where i, k index sectors. We report both weighted and unweighted coefficients, using the following formula to estimate the correlation coefficient (w_i are equal for all sectors in the unweighted version). In the weighted case, we adopt benchmark sector output levels as the weighting vector.

$$\rho = \frac{\sum_i \left[w_i \left(\Delta\hat{L}_i^E - \frac{\sum_k \Delta\hat{L}_k^E w_k}{\sum_k w_k} \right) \left(\Delta\hat{L}_i^S - \frac{\sum_k \Delta\hat{L}_k^S w_k}{\sum_k w_k} \right) \right]}{\sqrt{\left[\sum_i w_i \left(\Delta\hat{L}_i^E - \frac{\sum_k \Delta\hat{L}_k^E w_k}{w_k} \right)^2 \right] \left[\sum_i w_i \left(\Delta\hat{L}_i^S - \frac{\sum_k \Delta\hat{L}_k^S w_k}{w_k} \right)^2 \right]}}$$

The correlation between point estimates from the two modeling approaches is relatively high. The unweighted correlation is about 0.86, indicating a high degree of linear correspondence between the two approaches. When we apply weights to capture the relative size of sectors, correlation between the two approaches increases to between 0.95.

Finally, we conduct a regression analysis, in which we regress the predicted values from the CGE model on the predicted values from the econometric model. As above, we conduct both an unweighted regression and a weighted regression. Predicted values from each regression are shown in Figure 4 as dashed red and blue lines, and slope coefficients from each regression are given in Table 4. Figure 4 highlights the similarity of results derived from the regression model and CGE model. In particular, sectors that are more predicted to be more significantly impacted by the tax in the econometric model are generally also predicted to be more impacted by the tax in the CGE model. Table 4 shows that the slope coefficient from a regression of the CGE predicted values on the econometric predicted values is around 0.81 to 0.83, depending on whether coefficients are weighted, again indicating a close concordance between the two approaches. It is also worth noting that while the slope of the regression models is close to one, the intercept is somewhat below zero. This relates to the β_2 coefficient in (1). Unlike β_1, the variable relating to this coefficient varies at the province-time level, such that unobserved province-time heterogeneity could bias the coefficient estimate.

4 Decomposing model differences

Our comparison of the econometric and CGE models suggests that the two models predict similar - though not identical - patterns of sector-level employment response to a carbon tax. In this section we extend our analysis by using the models jointly to explore the validity of the assumptions in each. Specifically, we consider two categories of assumptions that affect results: the assumptions of the econometric model and the assumptions of the CGE model.

Our econometric strategy relies on a few key assumptions. First, we assume that industries in control provinces are unaffected by the introduction of the BC carbon tax. If industries are connected through interprovincial trade or factor markets, this may not be the case. Offsetting impacts in control industries (i.e. increases in employment, output or exports) in response to the tax could also cause our model to overestimate the treatment effect due to contamination of control units.

Second, the baseline specification of the econometric model also fails to control for differences in trade exposure across industries. All else equal, theory predicts that more trade-exposed industries should be more heavily impacted by the carbon tax. Leaving the role of trade exposure as the genesis of contamination effects aside, it is possible that — if a few trade-exposed industries are disproportionately impacted by the tax — they could skew the point estimates in our baseline model upward.

Third, our baseline econometric model also assumes that the impact of the carbon tax is linear in the carbon intensity of a sector of the economy. To the extent that this aspect of the model is misspecified, out point estimates could be biased.

Our CGE model also embodies a large number of assumptions. We explore the validity of the parametric assumptions, and do not test assumptions about the structure of the model. We focus in particular on assumptions related to the assumed production function as well as on trade elasticities, two important sources of uncertainty in the model.

4.1 Testing for contamination effects

The first test we perform is aimed at evaluating the tenability of the assumption that control industries are unaffected by the carbon tax (the so-called Stable Unit Treatment Value - or SUTVA - assumption). This is normally a maintained assumption in the type of empirical analysis we present here, and it is not straightforward to check for the validity of the assumption. In our case, we are able to use the CGE model to probe the likely validity of the SUTVA assumption (with the test obviously being conditional on the assumptions in the CGE model). To do this, we estimate our econometric model based on a pseudo-dataset constructed from the impacts of the tax *predicted* by the CGE model. The pseudo-dataset is constructed using the pre-treatment (2007) benchmark data used by the calibrated model combined with counterfactual impacts generated by the CGE model for each year that the tax is in place (2008-2012). Using the CGE model, we are then able to purge the data of any spillover effects of the tax in control provinces, and thus to determine whether the SUTVA assumption is likely to hold in our case.

Table 5 displays the results of our pseudo-regressions. The format of the tables mirrors the presentation of our baseline econometric results, and the empirical strategy and identification strategy is identical. Aside from the contents of the dataset used, there is a difference between the baseline regressions and the pseudo-regressions presented here. As previously noted, the level of aggregation is considerably higher in the CGE model than in the data used to estimate the econometric model. In this exercise, we use the more aggregate data generated by the CGE model.

Table 5 contains two columns. In the first column, we estimate (1) using the pseudo-data generated by the CGE model. The coefficients can be interpreted in the same way as in the above discussion. As expected based on the formal comparison of results above, we find coefficients of the same sign and very close to the same magnitude as when the original data is used. Specifically, sectors with higher carbon intensity are expected to cut employment more significantly when the carbon tax is imposed. In the second column, we estimate (1) using the pseudo-data generated from the CGE model, but purging any spillover effects of the carbon tax on counterfactual units – sectors and regions outside of BC. The coefficients change only very slightly in response to this change, suggesting that the SUTVA assumption is roughly maintained in the CGE model. To the extent that the CGE model is a valid representation of the real world, we would expect a similar spillover effect, and thus would expect the SUTVA assumption to also hold.

4.1.1 Alternative measures of emissions intensity

Our regression specification is based on difference-in-difference approach combined with a treatment intensity variable based on the carbon intensity of each sector. It is important to note, however, that theory does not provide clear guidance on how to construct the measure of carbon intensity that enters into (1). In the benchmark results, we construct the measure of carbon intensity by summing direct and indirect greenhouse gas emissions that are covered by the tax (i.e., those originating in British Columbia, and that are related to combustion) and dividing by the gross output of each sector. However, there are other possible ways of constructing this measure. For example, the numerator might include only direct carbon emissions, or the denominator might be sector value added (i.e., gross domestic product) rather than gross output.

Table 3 shows regression results using each of four possible combinations of our emission intensity variable in (1). Qualitatively, each of the four columns tells the same story – sectors with higher carbon emissions intensity experience larger losses in employment when the carbon tax is implemented, and sectors with low carbon intensity experience gains when the carbon tax is implemented (note that the magnitude of the Carbon × tax coefficient cannot directly be compared across specifications, since the carbon intensity measure is different in each specification). Coefficients are statistically significant in specification with gross output in the denominator, but not in specifications with GDP in the denominator.

We turn to the CGE model for guidance on the appropriate emission intensity measure to use. Figure 5 plots the employment results from the CGE model against the four different measures of emissions intensity. While each plot is suggestive of a linear relationship between sectoral GHG intensity and employment effects due to the carbon tax, measures using gross output as a denominator offer better predictive power (R-squared). The measure we use in our baseline specification - the indirect plus direct greenhouse gas emissions divided by gross output - offers the most explanatory power of the four specifications we evaluate.

4.1.2 Controlling for trade-exposure

One might expect the offsetting increases in output, employment and trade in control industries would be a problem primarily in goods that are heavily traded across provinces — so that when output is reduced in BC under the carbon tax, suppliers from other provinces can step in to satisfy demand. Here we explore an alternative regression in which we add interaction terms between our main carbon-tax treatment and the trade intensity of different sectors in the model. The resulting specification becomes:

$$\ln L_{ijrt} = \beta_1(EI_{jr} \times \tau_{rt}) + \beta_2\tau_{rt} + \beta_3(TI_{jr} \times \tau_{rt}) + \lambda^1_{ijr} + \lambda^2_{ijt} + \epsilon_{ijrt} \tag{9}$$

where TI_{jr} is defined as the share of output that is exported in sector j in province r in 2007.

With the addition of the trade-intensity interaction, the original two treatment terms (β_1 and β_2) now measure the impact of the carbon tax on sector employment conditional on trade exposure. If the contamination of control industries runs through trade — or if failing to control for trade exposure otherwise results in biased estimates of these coefficients — then examining the results of this regression may give an indication of the severity of these problems.

We report the regression results in the final column of Table 3. The coefficients on the original variables (β_1 and β_2 in (1)) are not markedly changed from the original estimates, which are presented in column (1), suggesting that the absence of trade exposure did not significantly bias the results. The coefficient on the trade intensity variable itself is not significantly different from zero, and is of unexpected sign (we would expect that a higher trade intensity would be associated with reduced employment in the presence of the tax; this is not what we found).

We obtain some support for the notion that sectoral trade intensity is not a strong determinant of the impact of the tax from similar estimation using the CGE pseudo-data. In particular, a regression including an interaction between the carbon tax and sector trade intensity (alongside other variables) does not yield an intuitively-signed or precisely-estiamted coefficient, and other coefficients magnitudes do not change as a result of the inclusion of the trade-intensity variable (column (3) of Table 5).

Based on these results, it appears that an econometric specification that omits sector trade intensity is likely appropriate. Interestingly, this is the same finding as reported in Martin et al. (2014). And importantly, it suggests that existing policies, which compensate industries for carbon pricing based partly on trade intensity (such as the EU and California), are likely not targeting compensation optimally.

4.2 Evaluating functional forms

Our baseline econometric model assumes that the impacts of the carbon tax treatment are linearly related to the carbon intensity of a sector's production technology. We explored the implications of controlling for trade-intensity in the previous sector but clearly there is scope for alternative functional forms, and again theory provides little guidance on the appropriate form. Here, we report briefly on tests that again use the CGE model to provide guidance on the appropriate specification of the econometric model. In particular, we run regressions on the pseudo-data generated from the CGE model that include higher-order polynomials in sectoral greenhouse gas intensity, and also test using logarithmic greenhouse gas intensity. In each case, the higher order terms are not significant, and so we retain the linear functional form.

We also estimate an alternative, non-parametric version of the econometric model with sector-specific treatment terms as shown in equation (10).

$$\ln L_{ijrt} = \sum_j (\text{Industry}_j \times \tau_{rt}) + \lambda + \epsilon_{ijrt} \tag{10}$$

where $Industry_j$ is an industry indicator variable. λ summarizes the array of fixed effects included in our baseline specification of the model.

The results of these regressions show similar effects to our baseline results. Figure 6 summarizes the results from the regressions on employment. While the standard errors are large due to larger data demands of the non-parametric specification, they also show a clear trend in the point estimates that mirrors our baseline results — with the most carbon-intensive industries seeing the largest declines in employment on a scale that is approximately linear with carbon intensity. Thus, our baseline specification seems justified.

4.3 Parameterizing the CGE model

Equations (2) through (8) describe the functional forms adopted for production and trade in the CGE model, which determine the sectoral responses to the carbon tax examined here. Choice of how to parametrize these functional forms can be key to determining the sectoral outputs. To determine the sensitivity of the results to changes in parametrization, we conduct two sensitivity analyses. First, we test the sensitivity of the results to changes in trade elasticities: σ^{D_i} and σ^{A_i}. We conduct one run where these elasticities are doubled from their initial levels, and another in which they are halved. Second, we test the sensitivity of the results to changes in the production function. The base parameterization of the model adopts a production function where capital and labour are combined in one nest, and this aggregate is then combined with an energy aggregate. Elasticities of substitution are from Okagawa and Ban (2008). Okagawa and Ban (2008) also estimate elasticities for a version in which capital and energy are in a nest, and this nest is combined with labour, and we test this alternative functional form with their estimated elasticities. Dissou et al. (2012) conduct similar estimation using Canadian data, for both of these nesting structures. In total, this gives four alternative sets of (sector-specific) production functions, and we run our model using each.

Figure 7 shows the sensitivity of the model results to differences in the trade elasticities, and Figure 8 shows the sensitivity of the model results to differences in the production function. The CGE model is somewhat sensitive to changes in the trade elasticities. In particular, for sectors in which a significant reduction in employment is predicted, the trade elasticity has an important effect. Higher trade elasticities reduce the ability of the sector to pass through costs of the carbon tax, and exacerbate losses in emissions intensive sectors. Differences are especially noteworthy in the cement (CEM) sector, and to a lesser degree in the oil refining (OIL), primary metal manufacturing (PRM), and transport (TRN) sectors. In contrast, the structure of the production function and the elasticities in the production function play very little role in determining the effect of the carbon tax on sectoral activity (at least, within the bounds of the combinations examined here).

Given the sensitivity of the CGE model to changes in the specification of trade elasticities, as well as the limited empirical basis for specifying these elasticities (recall that there is no data available on Canadian provincial trade elasticities), we attempt to use the results of the econometric model to verify and improve the specification of trade elasticities in the CGE model. In particular, we conduct simulations in the CGE model using alternative trade elasticity settings, and compare the results of these simulations to the econometric results, using the methods described earlier. We alter the trade elasticities in the model by multiplying the elasticities η, σ^{DM}, and σ^{PP} by a common multiplier (set at 1.0 in the original model specification). We show the results of this experiment in Figure 9. The original setting, described earlier, results in a R-squared value of 0.74 from a regression of the CGE results on the econometric results, and a slope coefficient of 0.83 from the same regression. It is clear that model fit improves somewhat when we increase the trade elasticities in the model. In particular, the slope coefficient is one when trade elasticities are set at 1.5 times their original value, and the overall model fit is highest when trade elasticities are set at about 1.3 times their original value.

5 Conclusions

Here we have compared ex ante estimates of the effects of the BC carbon tax based on a detailed CGE model of the Canadian economy with ex post estimates derived from a quasi-experimental econometric model. This allows us to test the theory and calibration underlying the CGE model as well as the potential for general equilibrium effects to undermine econometric tests based on the program evaluation paradigm. Overall, we find a strong degree of correspondence in the sign and relative magnitude of the sectoral impacts predicted by the two models and support from theory for our difference-in-difference design. This lends confidence to the notion that the CGE model is a useful tool for making ex ante predictions about the economic effects of environmental policies and for making welfare calculations. It also demonstrates how these models can play complementary roles in the evaluation of large-scale environmental policies — as a framework for model validation and for using statistical inference to deepen the empirical content of CGE analysis.

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6 Figures

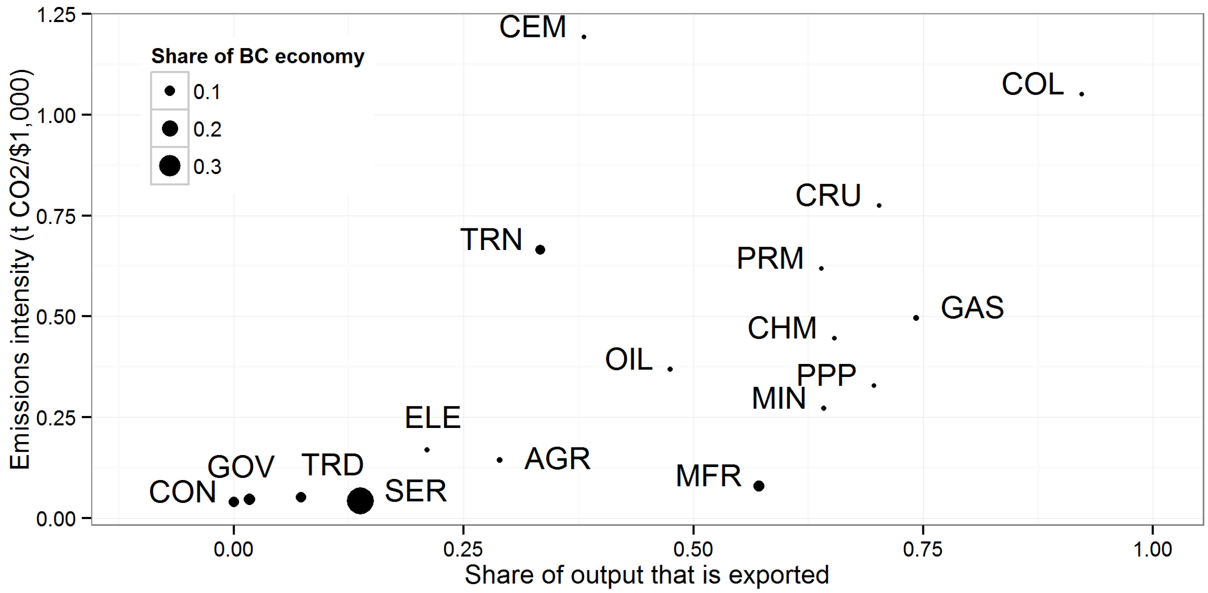

Figure 1: Benchmark CGE data. The emissions intensity is the direct and indirect emissions that are subject to the BC carbon tax (i.e., originating from fossil fuel combustion in British Columbia) divided by sector gross output. Area of circles corresponds to the size of each sector relative to the entire BC economy.

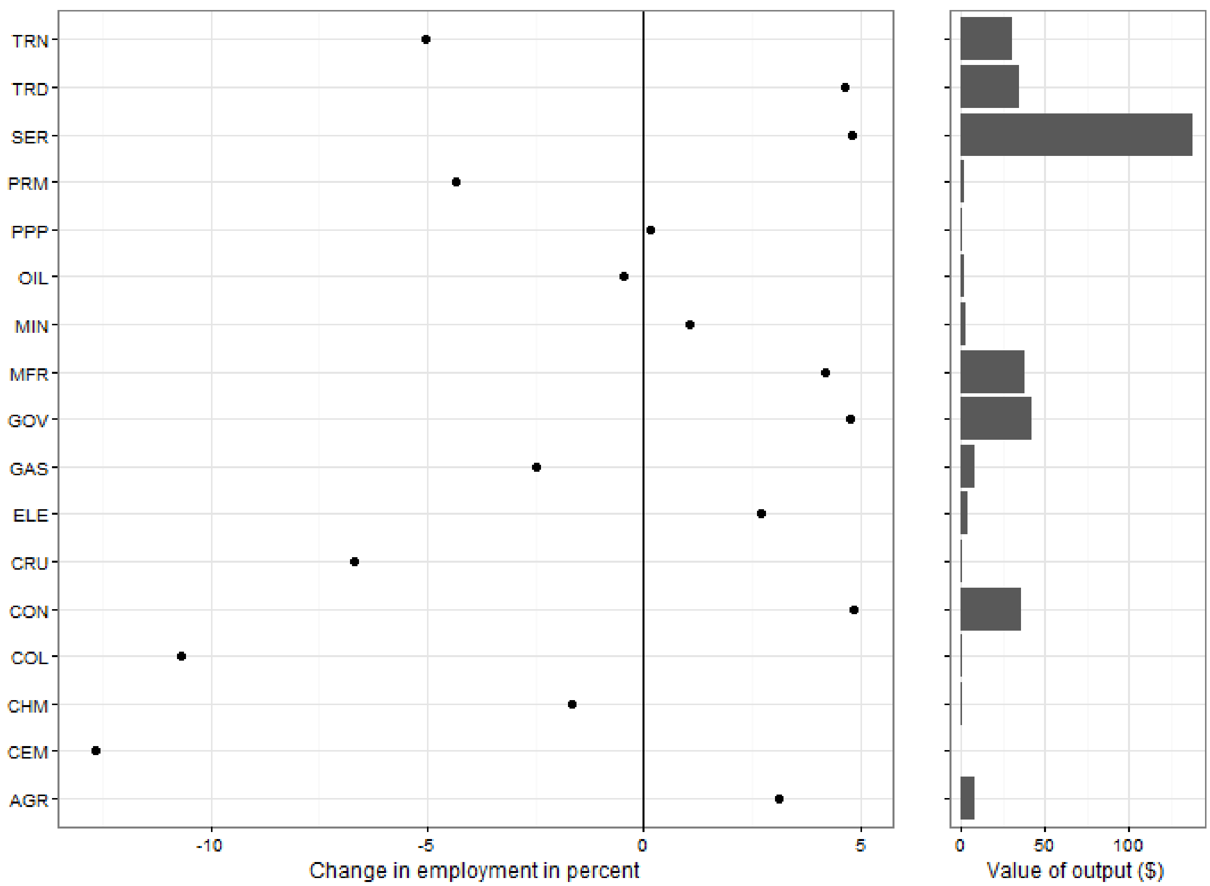

Figure 2: Left hand panel: Point estimates of predicted change in employment by sector due to a \$30/t CO_2 tax based on econometric estimates. Right hand panel: Total sector output in benchmark.

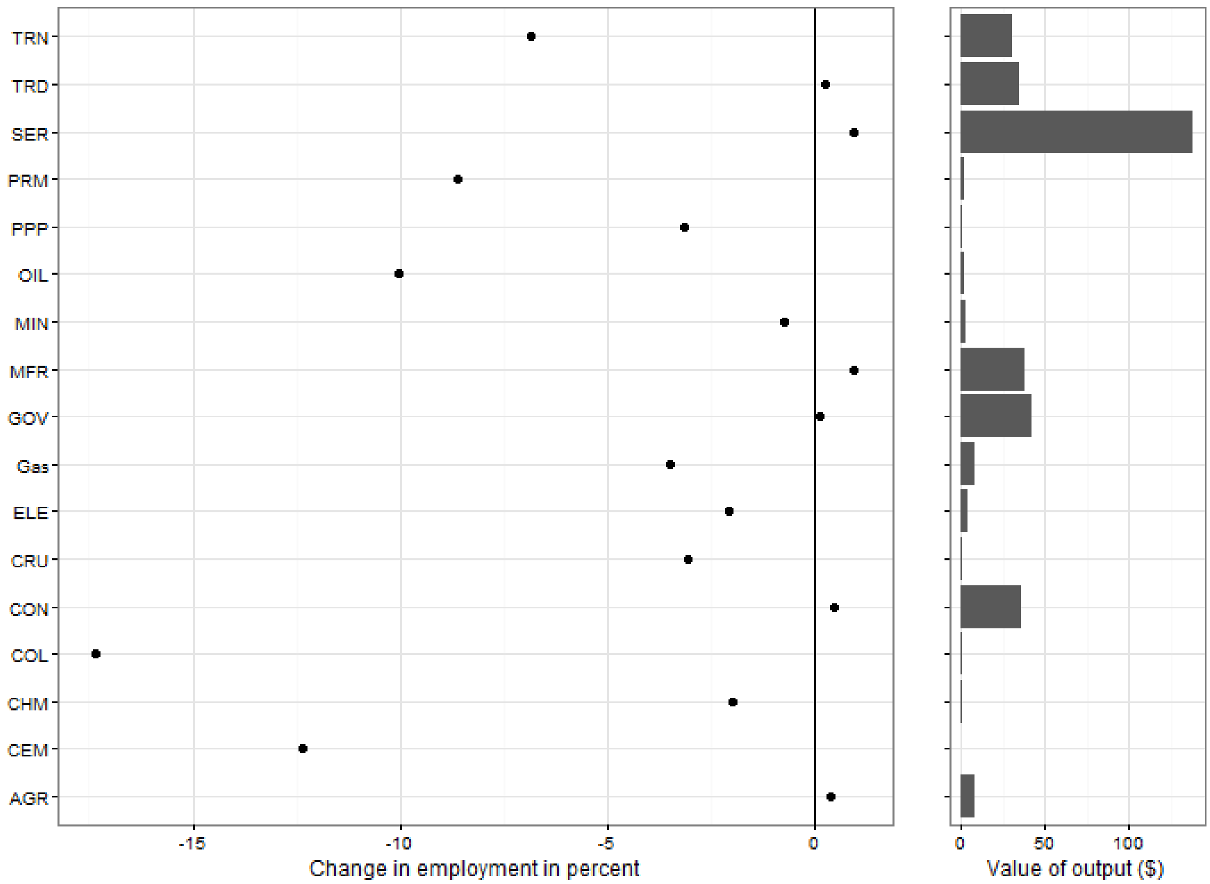

Figure 3: Left hand panel: Point estimates of predicted change in employment by sector due to a \$30/t CO_2 tax based on CGE model estimates. Right hand panel: Total sector output in benchmark.

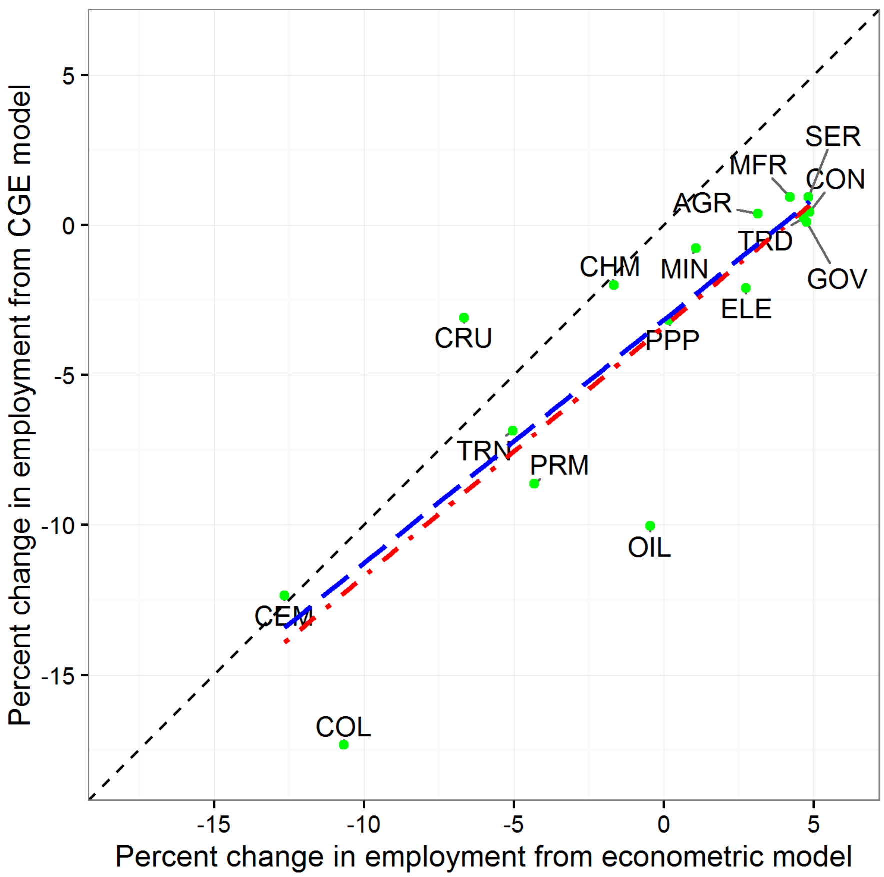

Figure 4: Comparison of econometric and CGE estimates of change in employment by sector associated with unilateral adoption of a \$30/t CO_2 tax in British Columbia. Dashed black line has intercept of 0 and slope of 1; points would fall along this line if the two models were identical. The red dash-dot line is the regression line from an unweighted regression of predicted values from the CGE model on predicted values from the econometric model. The blue long-dash line is the same thing, but using a weighted regression.

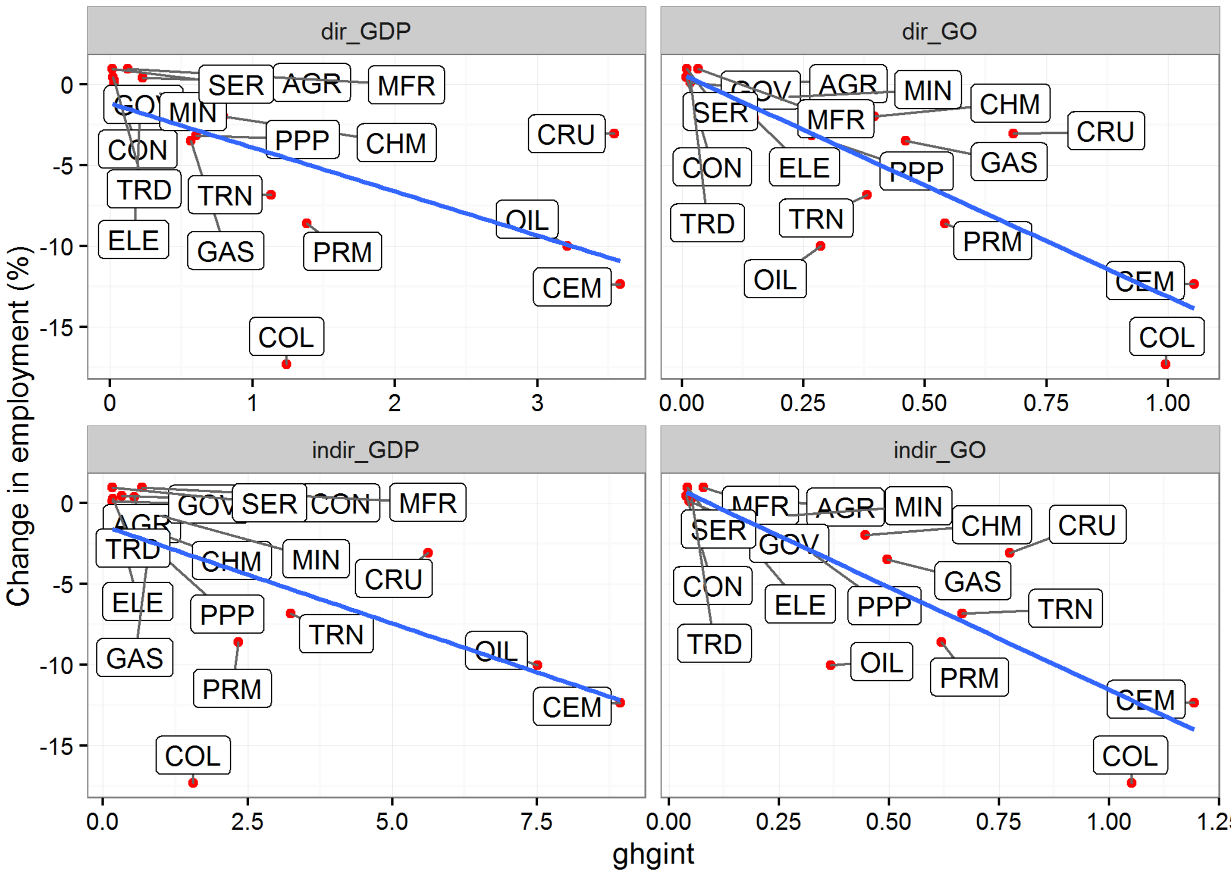

Figure 5: Results from CGE model for sectoral employment change in response to \$30/t tax, plotted against alternative measures of sector greenhouse gas intensity.

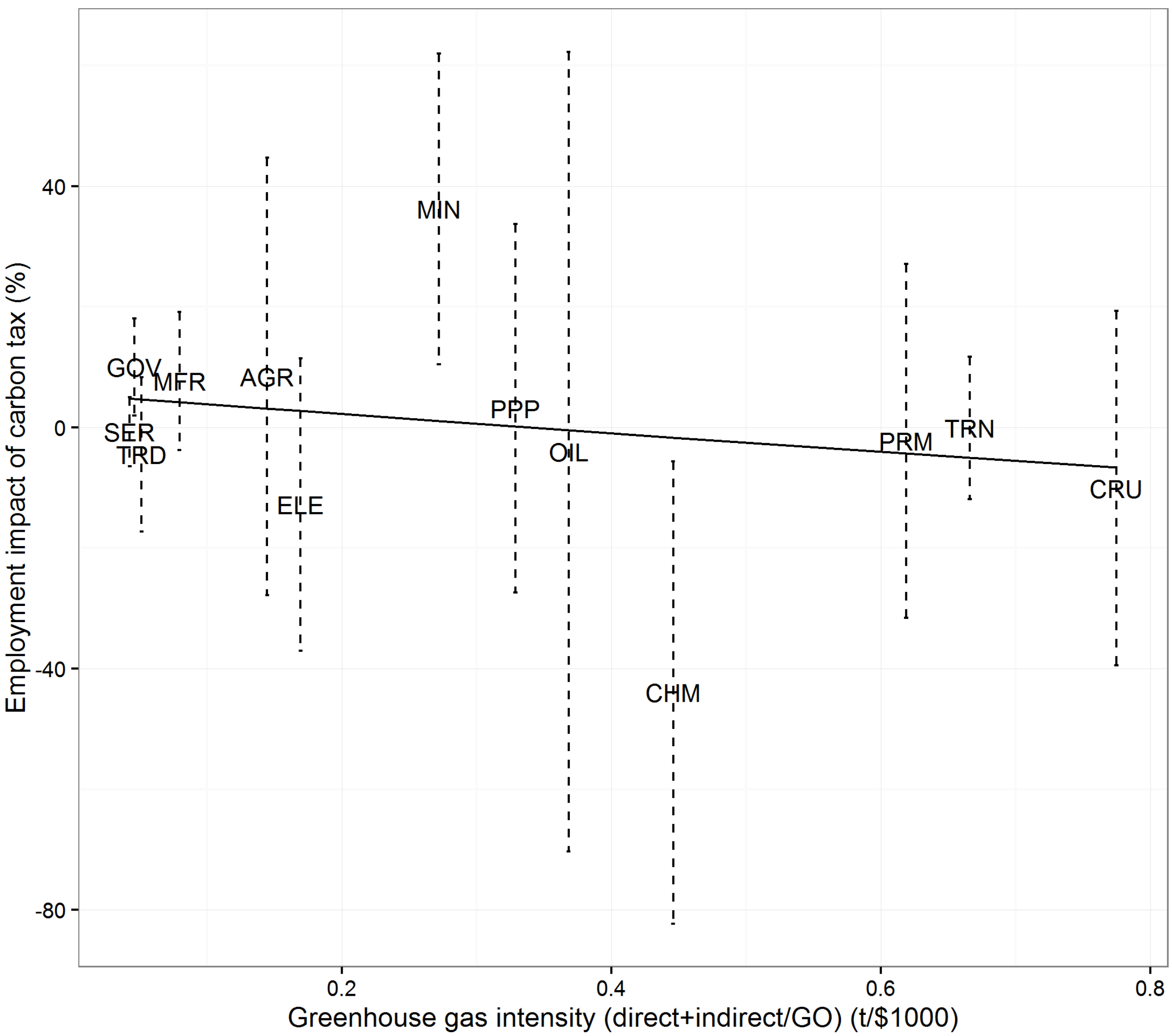

Figure 6: Results from alternative, non-parametric specification of the econometric model

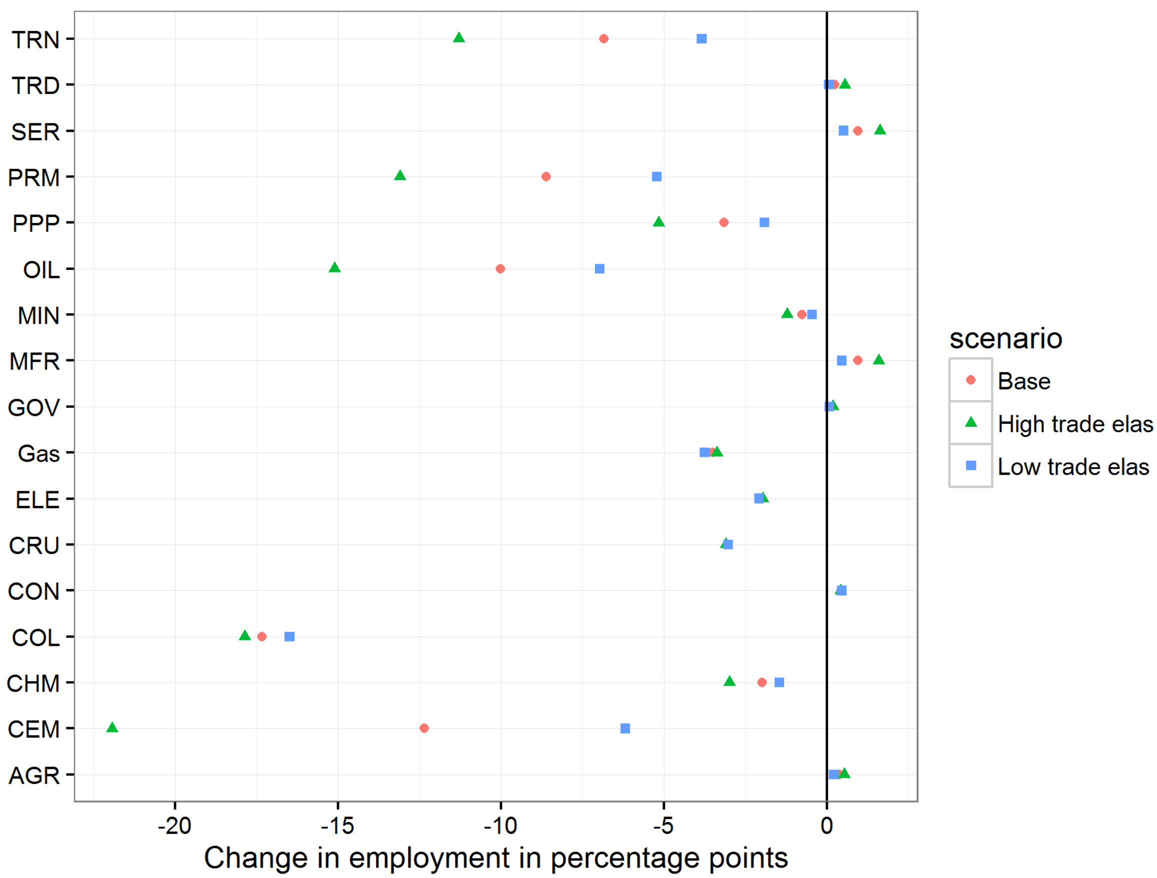

Figure 7: Sensitivity of CGE model results to differences in trade elasticity. In the High trade elas scenario, we double all trade elasticities; in the low trade elas scenario, we halve all trade elasticities.

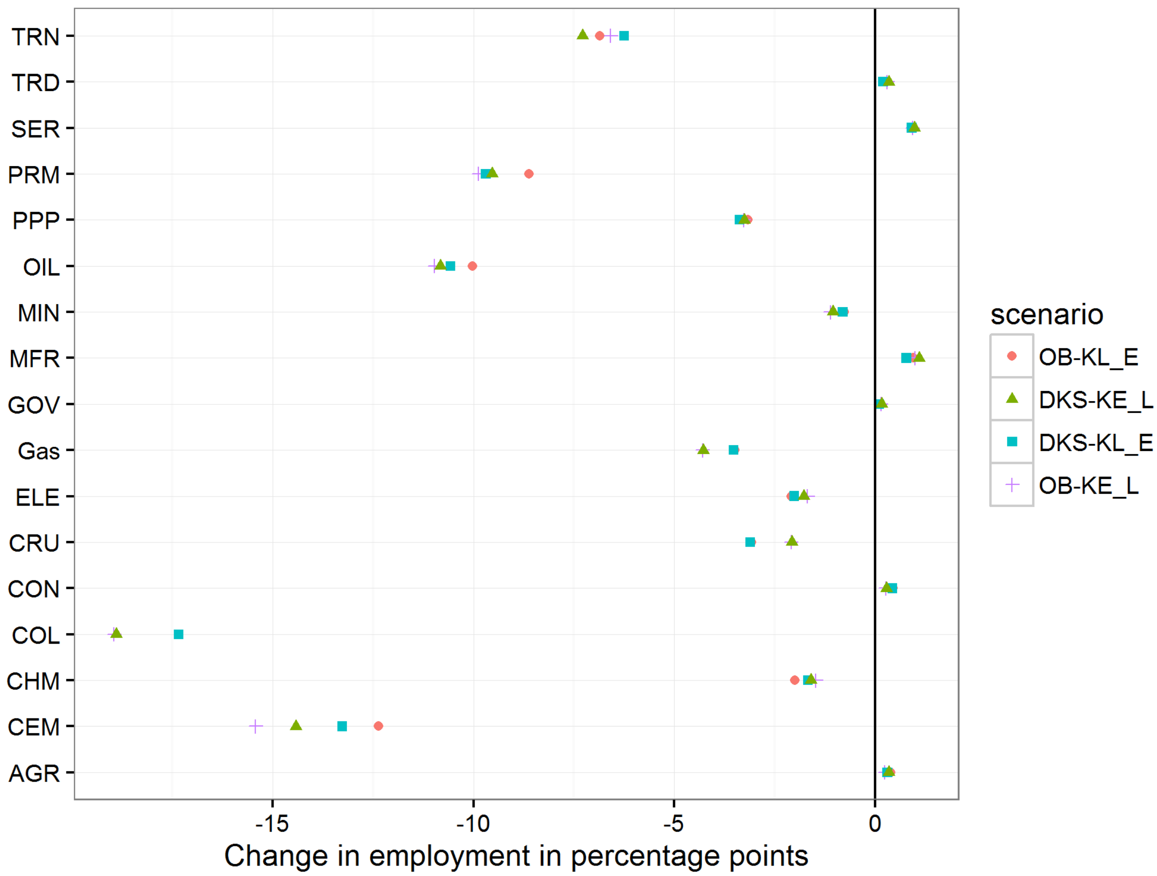

Figure 8: Sensitivity of CGE model results to differences in production nesting structure and elasticities. OB-KL_E is the base specification, taken from Okagawa and Ban (2008), where capital and labour are aggregated in a nest, and this nest is subsequently aggregated with energy. Points with a prefix of DKS are derived from Dissou et al. (2012).

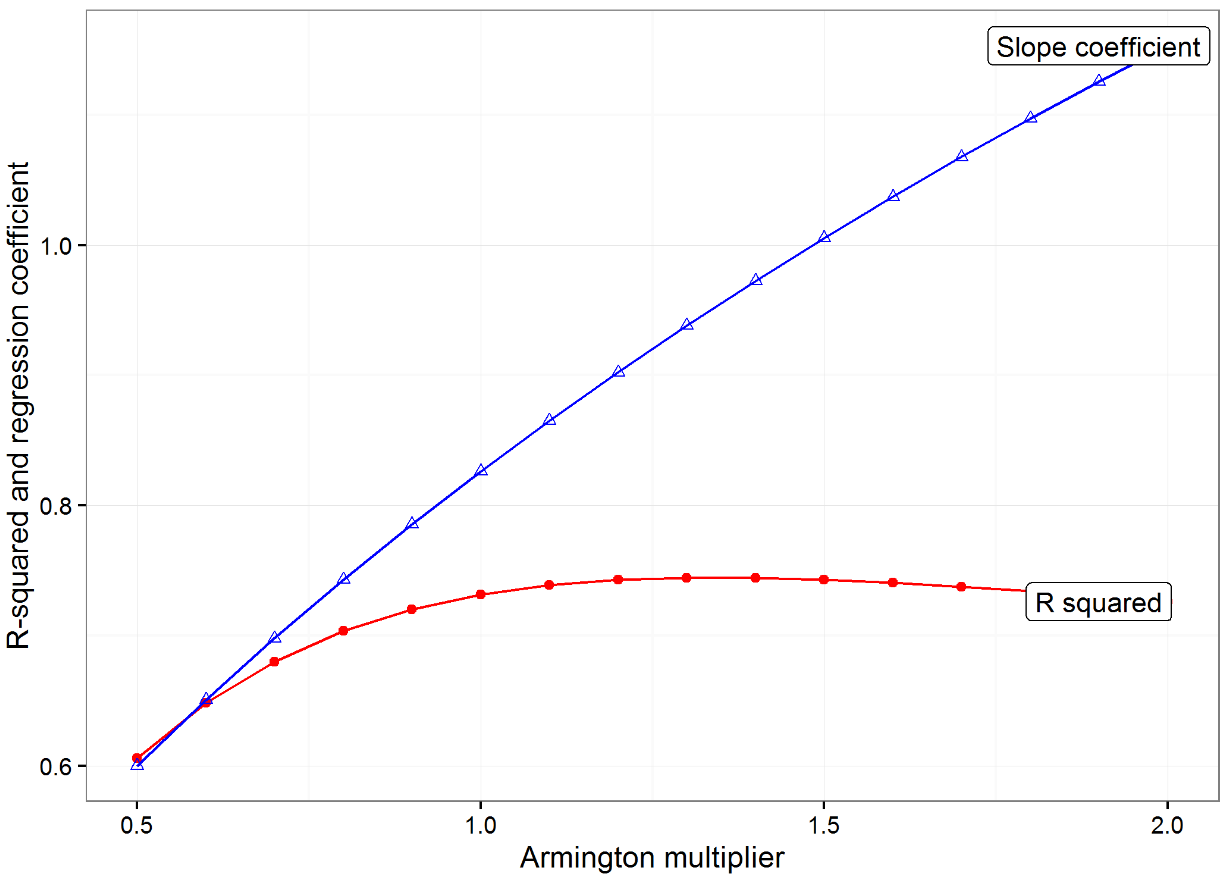

Figure 9: Comparison of CGE and econometric results with alternative values for trade elasticities.

7 Tables

Mnenomic	Description	Value of output ($B)	Value of exports ($B)	Emissions (Mt CO_2e)	Emission intensity (t/$1,000)	Export intensity
GAS	Natural gas	7.847	5.82	3.896	0.497	0.742
CRU	Crude oil	0.679	0.477	0.511	0.775	0.702
COL	Coal	1.163	1.072	1.202	1.051	0.922
OIL	Refined oil products	2.44	1.159	0.638	0.368	0.475
ELE	Electricity	4.349	0.914	0.657	0.169	0.210
AGR	Agriculture,fish,forests	8.197	2.366	0.857	0.144	0.289
MIN	Mining	2.762	1.774	0.626	0.272	0.642
CON	Construction	36.364	0.012	0.339	0.039	0.000
PPP	Pulp and paper	0.89	0.62	1.110	0.329	0.696
PRM	Primary metals	1.731	1.106	0.866	0.619	0.639
CHM	Chemicals	0.855	0.559	0.310	0.446	0.653
CEM	Cement	0.332	0.126	0.323	1.193	0.381
MFR	Other manufacturing	38.593	22.027	1.208	0.079	0.571
TRD	Retail and wholesale trade	35.358	2.592	0.627	0.051	0.073
TRN	Transport	30.77	10.235	12.127	0.666	0.333
SER	Services	138.169	19.068	1.463	0.042	0.138
GOV	Government	42.097	0.703	0.730	0.046	0.017

Table 1: Benchmark sector profiles for British Columbia from computable general equilibrium model. Emission intensity is calculated as the direct plus indirect emissions subject to the carbon tax (i.e., due to fossil fuel combustion and originating in British Columbia) divided by gross output. Export intensity is exports divided by gross output.

lnL	(1)	(2)	(3)	(4)	(5)
Carbon × Tax	-0.00109	-0.00309	-0.00354	-0.00528**	-0.00525**
	(0.00940)	(0.0129)	(0.0126)	(0.00235)	(0.00234)
Tax	-0.000606	0.00105	0.00213	0.00179	-0.00351*
	(0.00249)	(0.00229)	(0.00440)	(0.00111)	(0.00187)
Observations	4,181	4,181	4,181	4,181	4,181
R-squared	0.872	0.880	0.834	0.995	0.996
Time FE	Y				
Industry FE	Y				
Province FE	Y	Y			
Industry × time FE		Y	Y	Y	Y
Province trends			Y		Y
Industry × province FE				Y	Y

Standard errors clustered by province × industry are in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2: Econometric results, generated by estimating a regression of log employment on the carbon tax, the carbon tax interacted with the carbon intensity, and fixed effects as described in the table.

	(1)	(2)	(3)	(4)	(5)
EI Measure	(Dir + Indir)/GO	(Dir + Indir)/GDP	Dir/GO	Dir/GDP	(Dir+Indir)/GO
Carbon × Tax	-0.00528**	-0.000610	-0.00889**	-0.00182	-0.00510**
	(0.00235)	(0.000419)	(0.00434)	(0.00116)	(0.00235)
Trade × Tax					0.00205
					(0.00346)
Tax	0.00179	0.00150	0.00180*	0.00153	0.000561
	(0.00111)	(0.00112)	(0.00108)	(0.00106)	(0.00161)
Observations	4,181	4,181	4,181	4,181	4,181
R-squared	0.995	0.995	0.995	0.995	0.995

Standard errors clustered by province × industry are in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Robustness checks. Each model includes industry-time and industry-province fixed effects.

	unweighted	weighted
Sign concordance	0.81	0.98
Correlation	0.86	0.95
Linear regression	0.83	0.81

Table 4: Comparison between sector-level econometric and CGE predictions for the effect of a carbon tax. Weighted coefficients adopt benchmark sector output as weights.

VARIABLES	(1) lnL	(2) lnL	(3) lnL
Tax	0.0004**	0.0004*	0.0004
	(0.000)	(0.000)	(0.000)
Tax × intensity	-0.0046***	-0.0044***	-0.0045***
	(0.001)	(0.001)	(0.001)
Tax × trade intensity			0.0002
			(0.001)
Observations	888	888	888
R-squared	1.000	1.000	1.000
Year-sector FE	Y	Y	Y
Sector-region FE	Y	Y	Y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Pseudo-regression results using data generated from CGE model.

A Industry-sector concordance

Sector	Industry
AGR	Grains and other crop products
AGR	Live animals
AGR	Other farm products
CEM	Non-metallic mineral products
CHM	Chemical products
CON	Engineering construction
CON	Non-residential buildings
CON	Repair construction services
CON	Residential construction
CRU	Mineral and oil and gas exploration
CRU	Mineral fuels
ELE	Utilities
GOV	Education services provided by government sector
GOV	Health and social assistance services
GOV	Health services provided by government sector
GOV	Other aboriginal government services
GOV	Other federal government services
GOV	Other municipal government services
GOV	Other provincial and territorial government services
GOV	Sales of other government services
MFR	Alcoholic beverages and tobacco products
MFR	Computer and electronic products
MFR	Electrical equipment, appliances and components
MFR	Fabricated metallic products
MFR	Fish and seafood, live, fresh, chilled or frozen
MFR	Food and non-alcoholic beverages
MFR	Forestry products and services
MFR	Furniture and related products
MFR	Industrial machinery
MFR	Motor vehicle parts
MFR	Other manufactured products and custom work
MFR	Plastic and rubber products
MFR	Textile products, clothing, and products of leather and similar materials
MFR	Transportation equipment
MFR	Wood products
MIN	Metal ores and concentrates
MIN	Mineral support services
MIN	Non-metallic minerals
OIL	Refined petroleum products (except petrochemicals)
PPP	Wood pulp, paper and paper products and paper stock
PRM	Primary metallic products
SER	Accommodation and food services
SER	Administrative and support, head office, waste management and remediation services
SER	Arts, entertainment and recreation services
SER	Depository credit intermediation
SER	Education services
SER	Imputed rental of owner-occupied dwellings
SER	Information and cultural services
SER	Other finance and insurance
SER	Other services
SER	Printed products and services
SER	Professional services (except software and research and development)
SER	Published and recorded media products
SER	Real estate, rental and leasing and rights to non-financial intangible assets
SER	Research and development
SER	Sales of other services by Non-Profit Institutions Serving Households
SER	Services provided by Non-Profit Institutions Serving Households
SER	Software
SER	Support services related to farming and forestry
SER	Telecommunications
TRD	Retail margins, sales of used goods and commissions
TRD	Wholesale margins and commissions
TRN	Transportation and related services
TRN	Transportation margins

Table 6: Concordance of econometric and CGE sector definitions

B Model algebra

The model is formulated as a system of nonlinear inequalities. The inequalities correspond to the three classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero profit) conditions for constant-returns-to-scale producers, (ii) market clearance for all goods and factors and (iii) income-expenditure balances. The first class determines activity levels, the second class determines prices and the third class determines incomes. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint, a commodity price to a market clearance condition and an income to an income-expenditure balance.[13] Constraints on decision variables such as prices or activity levels allow for the representation of market failures and regulation measures. These constraints go along with specific complementary variables. In the case of price constraints, a rationing variable applies as soon as the price constraint becomes binding; in the case of quantity constraints, an endogenous tax or subsidy is introduced.[14]

In our algebraic exposition of equilibrium conditions below, we state the associated equilibrium variables in brackets. Furthermore, we use the notation Π_{gr}^{Z} to denote the unit profit function (calculated as the difference between unit revenue and unit cost) for constant-returns-to-scale production of item g in region r where Z is the name assigned to the associated production activity. Differentiating the unit profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling's Lemma), which appear subsequently in the market clearance conditions.

We use g as an index comprising all sectors/commodities including the final consumption composite, the public good composite and an aggregate investment good. The index r (aliased with s) denotes regions. The index EG represents the subset of all energy goods except for crude oil (here: coal, refined oil, gas, electricity) and the label X denotes the subset of fossil fuels (here: coal, crude oil, gas), whose production is subject to decreasing returns to scale given the fixed

[13]Due to non-satiation expenditure will exhaust income. Thus, the formal inequality of the income-expenditure balance will hold as an equality in equilibrium.

[14]An example for an explicit price constraint is a lower bound on the real wage to reflect a minimum wage rate; an example for an explicit quantity constraint is the specification of a (minimum)target level for the provision of public goods.

supply of fuel-specific factors. Tables 7 to 14 explain the notations for variables and parameters employed within our algebraic exposition. Figures 10 to 12 provide a graphical representation of the functional forms. Numerically, the model is implemented under GAMS (Brooke et al. 1996)[15] and solved using PATH (Dirkse and Ferris 1995)[16].

Zero profit conditions

1. Production of goods except for fossil fuels ($Y_{gr}|_{g \notin X}$):

$$
\begin{aligned}
\Pi^Y_{gr} = & \left(\theta^{EX}_{gr} \left(\frac{P^Y_{gr}(1 - tp^Y_{gr} - tf^Y_{gr})}{\bar{P}^Y_{gr}} \right)^{1+\eta} + \left(1 - \theta^{EX}_{gr}\right) \left(\frac{\mu(1 - tp^Y_{gr} - tf^Y_{gr})}{\bar{\mu}_{gr}} \right)^{1+\eta} \right)^{\frac{1}{1+\eta}} \\
& - \left(\theta^M_{gr} {P^M_{gr}}^{1-\sigma^M} + (1 - \theta^M_{gr}) \left(\left(\left(\theta^E_{gr} {P^E_{gr}}^{1-\sigma^E} + (1 - \theta^E_{gr}) \left(\theta^L_{gr} {P^L_r}^{1-\sigma^L} + (1 - \theta^L_{gr}) {P^K_{gr}}^{1-\sigma^L} \right)^{\frac{1}{1-\sigma^L}} \right)^{1-\sigma^E} \right)^{\frac{1}{1-\sigma^E}} \right)^{1-\sigma^M} \right)^{\frac{1}{1-\sigma^M}} \\
& \leq 0
\end{aligned}
$$

2. Production of fossil fuels ($Y_{gr}|_{g \in X}$):

$$
\begin{aligned}
\Pi^Y_{gr} = & \left(\theta^X_{gr} \left(\frac{P^Y_{gr}(1 - tp^Y_{gr} - tf^Y_{gr})}{\bar{P}^Y_{gr}} \right)^{1+\eta} + \left(1 - \theta^X_{gr}\right) \left(\frac{\mu(1 - tp^Y_{gr} - tf^Y_{gr})}{\bar{\mu}_{gr}} \right)^{1+\eta} \right)^{\frac{1}{1+\eta}} \\
& - \left(\theta^R_{gr} \left(\frac{P^R_{gr}(1 + tp^R_{gr} + tf^R_{gr})}{\bar{P}^R_{gr}} \right)^{1-\sigma^R_{gr}} + \left(1 - \theta^R_{gr}\right) \left(\theta^L_{gr} P^L_r + \sum_i \theta^R_{igr} \frac{(P^A_{ir}(1 + tp^D_{igr} + tf^D_{igr}) + a^{CO_2}_{igr} p^{CO_2}_r)}{\bar{P}^A_{igr}} \right)^{1-\sigma^R_{gr}} \right)^{\frac{1}{1-\sigma^R_{gr}}} \\
& \leq 0
\end{aligned}
$$

3. Sector-specific material aggregate (M_{gr}):

$$
\Pi^M_{gr} = P^M_{gr} - \left(\sum_{i \notin EG} \theta^M_{igr} \left(\frac{P^A_{ir}(1 + tp^D_{igr} + tf^D_{igr})}{\bar{P}^A_{igr}} \right)^{1-\sigma^D} \right)^{\frac{1}{1-\sigma^D}} \leq 0
$$

[15]Brooke, A., D. Kendrick and A. Meeraus (1996), *GAMS: A User's Guide*, Washington DC: GAMS

[16]Dirkse, S. and M. Ferris (1995), "The PATH Solver: A Non-monotone Stabilization Scheme for Mixed Complementarity Problems", *Optimization Methods & Software 5*, 123-156.

4. Sector-specific energy aggregate (E_{gr}):

$$\begin{aligned}\Pi^E_{gr} =& P^E_{gr} - \Bigg(\Bigg(\theta_{ELEgr}\left(\frac{P^A_{ELEr}(1+tp^D_{ELEgr}+tf^D_{ELEgr})}{\bar{P}_{ELEgr}}\right)^{1-\sigma^{ELE}} \\ &+ (1-\theta_{ELEgr})\Bigg(\Bigg(\theta_{COAgr}\left(\frac{P^A_{COAr}(1+tp^D_{COAgr}+tf^D_{COAgr})}{\bar{P}_{COAgr}} + a^{CO_2}_{COAgr}p^{CO_2}_r\right)^{1-\sigma^{COA}} \\ &+ (1-\theta_{COAgr})\Bigg(\theta_{OILgr}\left(\frac{P^A_{OILr}(1+tp^D_{OILgr}+tf^D_{OILgr})}{\bar{P}_{OILgr}} + a^{CO_2}_{OILgr}p^{CO_2}_r\right)^{1-\sigma^{OIL}} \\ &+ (1-\theta_{OILgr})\left(\frac{P^A_{GASr}(1+tp^D_{GASgr}+tf^D_{GASgr})}{\bar{P}_{GASgr}} + a^{CO_2}_{GASgr}p^{CO_2}_r\right)^{1-\sigma^{OIL}}\Bigg)^{\frac{1}{1-\sigma^{OIL}}}\Bigg)^{1-\sigma^{COA}}\Bigg)^{\frac{1}{1-\sigma^{COA}}}\Bigg)^{1-\sigma^{ELE}}\Bigg)^{\frac{1}{1-\sigma^{ELE}}} \\ \le& 0\end{aligned}$$

5. Armington aggregate (A_{ir}):

$$\Pi^A_{ir} = P^A_{ir} - \left(\left(\Theta^{DM}_{ir}\mu^{1-\sigma^{DM}} + \left(1-\Theta^{DM}_{ir}\right)\left(\sum_s \Theta^{MM}_{isr}P^{Y\,1-\sigma^{MM}_i}_{is}\right)^{\frac{1}{1-\sigma^{MM}}}\right)^{1-\sigma^{DM}}\right)^{\frac{1}{1-\sigma^{DM}}} \le 0$$

6. Labour supply (L_r):

$$\Pi^L_r = \frac{P^L_r\left(1-tp^L_r-tf^L_r\right)}{\overline{P^L_r}} - P^{LS}_r \le 0$$

7. Mobile capital supply (K):

$$\Pi^K = \left(\sum_r \Theta^K_r\left(\frac{P^K\left(1-tp^K_r-tf^K_r\right)}{\overline{P^K_r}}\right)^{1+\epsilon}\right)^{\frac{1}{1+\epsilon}} - P^{KM} \le 0$$

8. Welfare (W_r):

$$\Pi^W_r = P^W_r - \left(\Theta^{LS}_r P^{LS\,1-\sigma^{LS}_r}_r + \left(1-\Theta^{LS}_r\right)P^{Y\,1-\sigma^{LS}_r}_{Cr}\right)^{\frac{1}{1-\sigma^{LS}_r}} \le 0$$

Market clearance conditions

9. Labour (P^L_r):

$$L_r \ge \sum_g Y_{gr}\frac{\partial\Pi^Y_{gr}}{\partial P^L_r}$$

10. Leisure (P_r^{LS}):

$$\overline{L}_r - L_r \geq W_r \frac{\partial \Pi_r^W}{\partial P^{LS}}$$

11. Mobile capital (P^{KM}):

$$\sum_r \overline{KM}_r \geq K$$

12. Sector-specific capital (P_{gr}^K):

$$\overline{K}_{gr} + K \frac{\partial \Pi^K}{\partial P_{gr}^K} \geq \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial P_{gr}^K}$$

13. Fossil fuel resources ($P_{gr}^R|_{g \in X}$):

$$\overline{R}_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial (P_{gr}^R (1 + tp_{gr}^R + tf_{gr}^R))}$$

14. Energy composite (P_{gr}^E):

$$E_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial P_{gr}^E}$$

15. Material composite (P_{gr}^M):

$$M_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial P_{gr}^M}$$

16. Armington good (P_{ir}^A):

$$A_{ir} \geq \sum_g E_{gr} \frac{\partial \Pi_{gr}^E}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D) + a_{igr}^{CO_2} p_r^{CO_2})} + \sum_g M_{gr} \frac{\partial \Pi_{gr}^M}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D))}$$

17. Commodities (P_{ir}^Y):

$$Y_{ir} \frac{\partial \Pi_{ir}^Y}{\partial (p_{ir}^Y (1 - tp_{ir}^Y - tf_{ir}^Y))} \geq A_{ir} \frac{\partial \Pi_{ir}^A}{\partial P_{ir}^Y}$$

18. Private good consumption (P^Y_{Cr}):

$$Y_{Cr} \geq W_r \frac{\partial \Pi^W_r}{\partial P^Y_{Cr}}$$

19. Investment (P^Y_{Ir}):

$$Y_{Ir} \geq \bar{I}_r$$

20. Public Consumption (P^Y_{Gr}):

$$Y_{Gr} \geq \frac{INC^p_r}{P^Y_{Gr}} + \theta^G_r \frac{INC^f}{P^Y_{Gr}}$$

21. Welfare (P^W_r):

$$W_r \geq \frac{INC^{RA}}{P^W_r}$$

22. Carbon emissions (P^{CO}_2):

$$\overline{CO_2} \geq \sum_r \sum_{i \in EG} \sum_g E_{gr} \frac{\partial \Pi^E_{gr}}{\partial (P^A_{ir}(1 + tp^D_{igr} + tf^D_{igr}) + a^{CO_2}_{igr} p^{CO_2}_r)}$$

Income-expenditure balances

23. Income of representative consumer (INC^{RA}_r):

$$\begin{aligned} INC^{RA}_r = {} & P^{LS}_r \overline{L}_r \\ & + \sum_{x \in g} P^R_{gr} \bar{R}_{gr} \\ & + P^{KM} \overline{KM}_r \\ & + \sum_g P^K_{gr} \overline{K}_{gr} \\ & - P^Y_{Ir} \bar{I}_r \\ & + p^{CO_2}_r \theta^{CO_2}_r \overline{CO_2} \\ & + \mu \overline{BOP}^{RA}_r \\ & - \chi_r \mu \\ & - \varepsilon_r P^Y_{Cr} \end{aligned}$$

24. Income of provincial government (INC_r^p):

$$
\begin{aligned}
INC_r^p = {} & L_r\, P_r^L\, tp_r^L \\
& + \sum_{g \in x} \bar{R}_{gr}\, P_{gr}^R\, tp_{gr}^R \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial P_{gr}^K}\, P_{gr}^K\, tp_r^K \\
& + \sum_i \sum_g \Bigg(E_{gr} \frac{\partial \Pi_{gr}^E}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D) + a_{igr}^{CO_2}\, p_r^{CO_2})}\, P_{ir}^A\, tp_{igr}^D \\
& \qquad + M_{gr} \frac{\partial \Pi_{gr}^M}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D))}\, P_{ir}^A\, tp_{igr}^D \Bigg) \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial (p_{gr}^Y (1 - tp_{gr}^Y - tf_{gr}^Y))}\, P_{gr}^Y tp_{gr}^Y \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial (\mu (1 - tp_{gr}^Y - tf_{gr}^Y))}\, \mu tp_{gr}^Y \\
& + \mu \overline{BOP}_r^p \\
& + \chi_r \mu
\end{aligned}
$$

25. Income of federal government (INC^f):

$$
\begin{aligned}
INC^f = {} & \sum_r \Big(L_r\, P_r^L\, tf_r^L \\
& + \sum_{g \in x} \bar{R}_{gr}\, P_{gr}^R\, tf_{gr}^R \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial P_{gr}^K}\, P_{gr}^K\, tf_r^K \\
& + \sum_i \sum_g \Bigg(E_{gr} \frac{\partial \Pi_{gr}^E}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D) + a_{igr}^{CO_2}\, p_r^{CO_2})}\, P_{ir}^A\, tf_{igr}^D \\
& \qquad + M_{gr} \frac{\partial \Pi_{gr}^M}{\partial (P_{ir}^A (1 + tp_{igr}^D + tf_{igr}^D))}\, P_{ir}^A\, tf_{igr}^D \Bigg) \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial (p_{gr}^Y (1 - tp_{gr}^Y - tf_{gr}^Y))}\, P_{gr}^Y tf_{gr}^Y \\
& + \sum_g Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial (\mu (1 - tp_{gr}^Y - tf_{gr}^Y))}\, \mu tf_{gr}^Y \\
& + \mu \overline{BOP}^f \\
& + \varepsilon_r\, P_{cr}^Y \Big)
\end{aligned}
$$

26. Equal-yield for provincial government demand (χ_r):

$$\frac{INC_r^P}{P_{Gr}^Y} \geq \bar{G}_r^P$$

27. Equal-yield for federal government demand (ε):

$$\sum_r \theta_r^G \frac{INC^f}{P_{Gr}^Y} \geq \sum_r \bar{G}_r^f$$

B.1 Notation

Symbol	Description
i	Goods excluding final demand goods
g	Goods including intermediate goods ($g = i$) and final demand goods, i.e. private consumption ($g = C$), investment ($g = I$) and public consumption ($g = G$)
r (alias s)	Regions
EG	Energy goods: coal, refined oil, gas and electricity
X	Fossil fuels: coal, crude oil and gas

Table 7: Sets

Symbol	Description
Y_{gr}	Production of good g in region r
E_{gr}	Production of energy composite for good g in region r
M_{gr}	Production of material aggregate for good g in region r
A_{ir}	Production of Armington good i in region r
L_r	Labour supply in region r
K	Capital supply
W_r	Production of composite welfare good

Table 8: Activity variables

Symbol	Description
p_{gr}^Y	Price of good g in region r
p_{gr}^E	Price of energy composite for good g in region r
p_{gr}^M	Price of material composite for good g in region r
p_{ir}^A	Price of Armington good i in region r
p_r^L	Price of labour (wage rate) in region r
p_r^{LS}	Price of leisure in region r
P_{gr}^K	Price of capital services (rental rate) in sector g and region r
p_{gr}^R	Rent to fossil fuel resources in fuel production in sector g ($g \in X$)and region r
$p_r^{CO_2}$	CO_2 price in region r
p^{KM}	Price of interregionally mobile capital
p_{gr}^K	Price of sector-sector specific capital
p_r^W	Price of composite welfare (utility) good
μ	Exchange rate

Table 9: Price variables

Symbol	Description
INC_r^{RA}	Income of representative agent in region r
INC_r^{p}	Income of provincial government in region r
INC^{f}	Income of federal government

Table 10: Income variables

Symbol	Description
tp_{gr}^{Y}	Provincial taxes on output in sector g and region r
tf_{gr}^{Y}	Federal taxes on output in sector g and region r
tp_{gr}^{R}	Provincial taxes on resource extraction in sector g and region r
tf_{gr}^{R}	Federal taxes on resource extraction in sector g and region r
tp_{igr}^{D}	Provincial taxes on intermediate good i in sector g and region$r$
tf_{igr}^{D}	Federal taxes on intermediate good i in sector g and region$r$
tp_{r}^{L}	Provincial taxes on labour in region r
tf_{r}^{L}	Federal taxes on labour in region r
tp_{r}^{K}	Provincial taxes on capital in region r
tf_{r}^{K}	Federal taxes on capital in region r
$\bar{P}_{gr}^{Y}$	Reference price of good g in region r
$\bar{\mu}_{gr}$	Reference value of exchange rate
$\bar{P}_{gr}^{R}$	Reference price of fossil fuel resource g in region r
$\bar{P}_{ir}^{A}$	Reference price of Armington good i in region r
$\bar{P}_{r}^{L}$	Reference price of labour (wage rate) in region r
$\bar{P}_{r}^{K}$	Reference price of capital in region r

Table 11: Tax rates and reference prices

Symbol	Description
θ_{gr}^{EX}	Value share of international market exports in domestic production of good g in region r
θ_{gr}^{E}	Value share of energy in the production of good g in region r
θ_{gr}^{M}	Value share of the material aggregate within the composite of value-added and material in the production of good g in region r
θ_{gr}^{L}	Value share of labour in the value-added composite of good g production in region r
θ_{gr}^{R}	Value share of fossil fuel resource in fossil fuel production ($g \in X$) in region r
θ_{gr}^{ELE}	Value share of electricity in the energy composite of good g production in region r
θ_{gr}^{COA}	Value share of coal in the coal-oil-gas composite of good g production in region r
θ_{gr}^{OIL}	Value share of oil in the oil-gas composite of good g production in region r
θ_{ir}^{DM}	Value share of domestically produced inputs to Armington production of good g in region r
θ_{isr}^{MM}	Value share of imports from region s in the import composite of good i to region r
θ_{r}^{K}	Value share of capital supply to region r in overall (mobile) capital supply
θ_{r}^{LS}	Value share of leisure demand in region r
θ_{r}^{G}	Share of region r in overall public good consumption
$\theta_{r}^{CO_2}$	Share of region r in overall CO_2 emission endowment

Table 12: Cost shares

Symbol	Description
$\overline{L}_r$	Aggregate time (labour and leisure) endowment of region r
$\overline{K}_{gr}$	Sector-specific capital endowment of region r
$\overline{R}_{gr}$	Endowment of fossil fuel resource g by region r $(g \in X)$
$\overline{BOP}_r^{RA}$	Representative agent's balance of payment deficit or surplus in region r
$\overline{BOP}_r^{p}$	Provincial government's balance of payment deficit or surplus in region r
$\overline{BOP}^{f}$	Federal government's initial balance of payment deficit or surplus
$\overline{CO_2}$	Endowment with carbon emission rights
$a_{igr}^{CO_2}$	Carbon emissions coefficient for fossil fuel i $(i \in X)$in good g production of region r
$\overline{I}$	Exogenous investment demand
G_r^p	Exogenous provincial government demand
G_r^f	Exogenous federal government demand

Table 13: Endowments and emissions coefficients

Symbol	Description
χ_r	Lump-sum transfers to warrant equal-yield constraint for provincial government r
ε_r	Lump-sum transfers to warrant equal-yield for federal government

Table 14: Additional variables

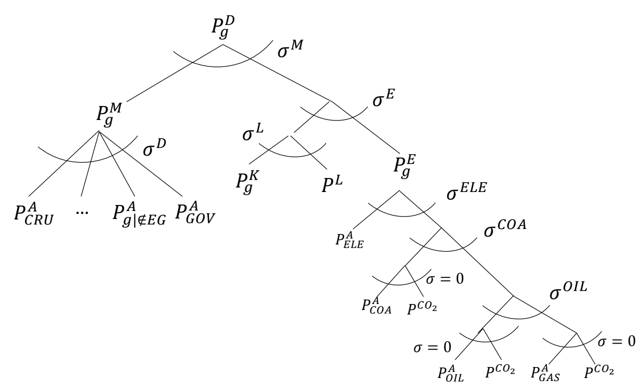

Figure 10: Cost function for non-fossil fuel sectors (region (r) subscripts dropped to reduce notational clutter.)

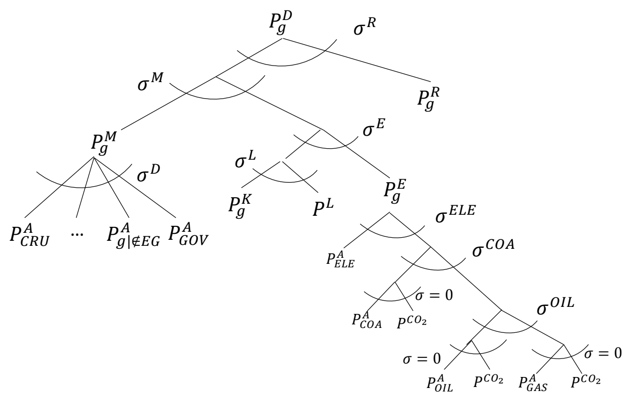

Figure 11: Cost function for fossil fuel sectors (region (r) subscripts dropped to reduce notational clutter.)

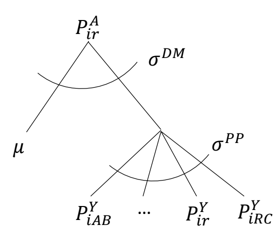

Figure 12: Cost function for Armington good i in region r

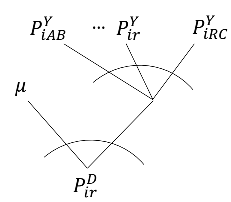

Figure 13: Transformation of output of good i in region r