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ABSTRACT
This paper develops GARCH and VEC-MGARCH-based tests of four hypotheses from Fama and French (1988) involving linkages between spot and futures prices — both their levels and variances. The tests are applied to monthly data for seven metals traded on the London Metal Exchange over the period 1988:11, where available, through 2008:07.

JEL **classifications: C21, C32, G13, Q39**

Keywords: Samuelson Futures Price Hypothesis, Cost-of-Carry Model, Theory of Storage, Cointegration, MGARCH Models, Spot and Futures Prices

Introduction

Movements in the spot and futures prices for primary commodities traded on formal exchanges are of key interest to portfolio managers, producers, consumers, hedgers and speculators. This paper analyzes the linkage between future and spot prices for seven mining products traded on the London Metal Exchange (LME), taking into account the relationship between the quantities held on inventories of these products and the variability of their prices.

For years, the theory of storage or "cost-of-carry model," originally proposed by Working (1933) and Kaldor (1939), has been the framework of analysis for the linkage between spot and futures prices. The model states that the spread between spot and futures prices is determined by fundamental supply-and-demand conditions and is related to storage costs, inventory levels and convenience yields. Empirical work by Fama and French (1988) finds considerable support for the theory when studying precious metals such as gold and silver, and base metals such as copper, zinc and aluminum.

Market participants are concerned with trend movements and the difference between futures and spot prices (the 'basis'). Relative volatilities are also of paramount importance. The famous Samuelson hypothesis (1965) provides a starting point for discussions involving volatility: Futures prices vary less than spot prices, and the variation of futures prices is a decreasing function of maturity. Fama and French (1988: 1075) develop an interesting refinement of the much-tested Samuelson's proposition: "futures prices are less volatile than spot prices (meaning that Samuelson hypothesis holds) when inventory is low. When inventory is high, however, the theory predicts that spot and futures prices have roughly the same variability."

Much of the Fama-French analysis focuses on the interest adjusted basis (*IAB*):

$$IAB_{t,t+T} \equiv \log F_{t,t+T} - \log P_t - i_t * T, \tag{1.1}$$

where P_t is the spot price at time t, $F_{t,t+T}$ is the futures price at t for delivery at $t + T$, and $i_t \equiv \log(1 + i_{TB,t})$ is the continuously compounded (short term) interest rate on T-bills over the maturity of the futures contract. They highlight a number of testable implications, which we dub the Fama-French-Samuelson (FFS) hypotheses:

- Hypothesis I (the "cost of carry" proposition): Interest-adjusted basis is a positive concave function of the level of inventories (Fama and French 1988: 1077).

- Hypothesis II: When the interest-adjusted basis is negative (indicating that inventory is "low" or the market is 'tight"), the interest-adjusted basis is more variable (Fama and French 1988: 1092).

- Hypothesis III (The Samuelson Hypothesis): Future prices vary less than spot prices.

- Hypothesis IV (The Fama-French Hypothesis): When inventory is low, there is more independent variation in spot and future prices, whereas when inventory is high, future and spot prices are almost perfectly correlated.

In their empirical work, Fama and French (FF) use the following dummy variable to measure market tightness:

$$D_t = \begin{cases} 1 & IAB_t < 0 \\ 0 & otherwise, \end{cases} \tag{1.2}$$

where the subscript notation on *IAB* from (1.1) has been simplified by leaving the $t + T$ implicit. They provide the following justification for this dummy variable:

> A natural approach would be to test directly the hypotheses about inventory and the variation of spot and futures prices. The metals we study are produced and consumed internationally, however, and the accuracy of data of short-term variation in aggregate inventory is questionable. We use a proxy for inventory suggested by the storage

> equation [our equation (1.7) below]. When inventory is high, the marginal convenience yield on inventory is low and the interest-adjusted basis is positive. When inventory is low, the convenience yield is high and the interest-adjusted basis is negative. Thus, the sign of the interest-adjusted basis is a proxy for high (+) and low (-) inventory. (Fama and French 1988: 1079)

Although we consider the Fama-French market tightness dummy, we also use data on inventories at LME warehouses around the world as a proxy for market conditions. While not a comprehensive measure of all inventories held by producers, intermediaries, traders and final consumers, LME stocks are presumably a reasonable (inverse) proxy for global market tightness. It is not clear *a priori* which proxy is preferable. However, it is clear that some of the FFS hypotheses are about spot price-futures price linkages in levels, while others pertain to variances and covariance (or correlation).

Some recent works (e.g., Geman and Smith 2012; Symeonidis, Prokopczuk, Brooks and Lazar 2012) evaluate testable hypotheses of the theory of storage for mineral, energy and agricultural commodities using traditional statistical techniques such a correlation and regression analysis. They do not, however, properly control for econometric issues such as non-stationarity and time-varying volatilities present in commodity prices. This paper develops a number of tests of the FFS hypotheses using both univariate GARCH and vector error correction - multivariate GARCH (MGARCH) models of conditional volatility and conditional correlation. We apply these tests to seven industrial metals traded on the London Metal Exchange (LME).

The rest of the paper goes as follows. Section I briefly reviews the cost-of-carry model (or theory of storage), which describes a hypothesized relationship between the *IAB* and the level of inventory for the commodities in question. Section II provides striking graphical evidence on Hypothesis I (the concave relationship between *IAB* and inventory) and Hypothesis II (the dependence of the ***variance*** of *IAB* on the level of inventory holdings) using nonparametric kernel estimation. Section III carries out unit root and cointegration tests that are precursors to the model estimation that follows. Section IV presents the results of the estimation of cost-of-carry models relating *IAB* to *LINV* with GARCH processes that depend on different measures of market tightness. The results provide strong empirical support for Hypotheses I and II. Section V estimates bivariate error correction models for futures and spot prices, taking into account the long-run relationship among *LF*, *LP*, *i* and *LINV* in levels – as implied by the COC model – as well as the multivariate GARCH nature of their respective error processes. A number of hypothesis tests based on these VEC-MGARCH models provide strong support for our generalized Samuelson's hypothesis (III), and weaker support for the Fama-French hypothesis (IV). Section VI presents conclusions and final comments.

I Conceptual Framework: Cost-of-Carry Model

The theory of storage relates the spot and futures prices for a commodity by comparing two strategies for securing a unit of the commodity *T* periods in the future.

Strategy 1: Purchase a futures contract at time *t* for delivery of one unit of metal at $t + T$ at price $F_{t,t+T}$. As funds are not needed until the contract matures, they earn (continuously compounded) interest in the meantime, *i*. Thus, the present value cost of this strategy for obtaining one unit of the commodity at $t + T$ equals:

$$PV_1 = e^{-iT} F_{t,t+T}. \qquad (1.3)$$

Strategy 2: Purchase the commodity in the spot market at price P_t and hold until $t + T$. Under this strategy the purchaser will have to pay warehousing (storage and insurance) costs, which are assumed to equal w per dollar of inventory per period. The present value of strategy 2 equals:

$$PV_2 = P_t e^{wT}. \tag{1.4}$$

In the simplest model of equilibrium (to be generalized below to include convenience yields), the present values of the two strategies should be equal so as to avoid arbitrage opportunities. This implies that:

$$F_{t,t+T} = P_t e^{(i+w)T}, \tag{1.5}$$

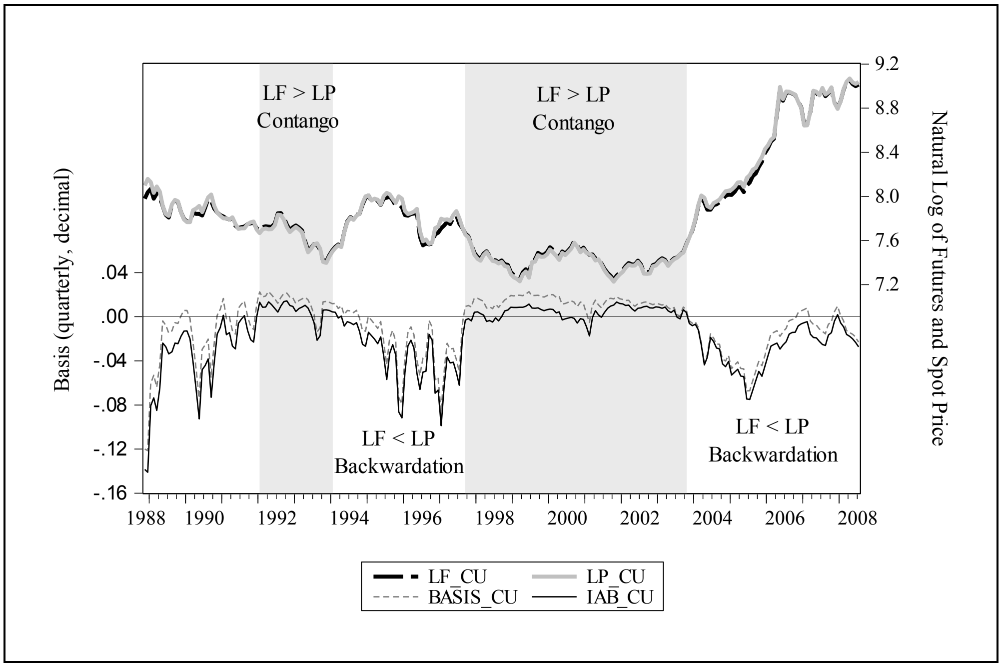
or in natural logarithmic terms where the *IAB* is in square brackets:

$$[\log F_{t,t+T} - \log P_t - i_t * T] - w_t * T = 0 \tag{1.6}$$

This equilibrium has implications that are not supported by the data for most commodities: (1) the *IAB* should *always* be non-negative (assuming warehousing and insurance costs are non-negative), and (2) the futures price should *always* lie above the spot price. That is, the market is always in contango.

Empirically, however, we observe that the *IAB* is sometimes positive and sometimes negative; commodity markets are sometime in contango and other times in backwardation. Figure 1 for the case of copper is illustrative.

Figure 1: Futures and Spot Prices, Basis, and IAB for LME Copper*



* LF: log of the futures price. LP: log of the spot price. CU: Copper. IAB: Interest Adjusted Basis.

To explain backwardation, the theory of storage articulated by Working (1948, 1949), Telser (1958), Brennan (1958, 1991), and Williams (1986) introduces a 'convenience yield' from holding inventory (i.e. a spot position).[1,2] The convenience yield is assumed to be a convex function of the level of inventory holdings, $c_t = c(INV_t)$ where $c' > 0$ *and* $c'' > 0$, reflecting the diminishing marginal productivity of inventory.[3] With the addition of the convenience yield, *c*, the equilibrium condition (1.6) expressed in logarithms becomes:

[1] This might reflect the direct utility benefit of holding and admiring your commodity inventory (plausible for gold, but perhaps not for tin) or the 'option value' of being able to consume your inventory before the end of the period in an emergency.
[2] Brennan (1958) generalizes the theory presented here to allow for a risk premium for those holding spot positions. In this case, the cost-of-carry function is concave at low and moderate inventory levels, but may become convex at high inventory levels.
[3] More recent work of Ramey (1989) argues that inventories of raw materials, intermediate goods, and final goods are all appropriately viewed as factors of production. The convenience yield can, in this context, be interpreted as the marginal productivity of inventory.

$$[\log F_{t,t+T} - \log P_t - i_t * T] - w_t * T - c(INV_t) * T = 0. \quad (1.7)$$

From (1.7) it is clear that with the introduction of the convenience yield the future rate may lie above or below the spot rate (contango or backwardation). Thus, the *IAB* may be positive or negative. Equation (1.7) is the cost-of-carry model that predicts a concave relationship between *IAB* and *INV*. When estimating the COC model below, the convex convenience yield relationship is captured using a log specification:[4]

$$c_t = \delta_0 + \delta_1 \log(INV_t). \quad (1.8)$$

Thus, the log form of the long-run equilibrium condition equals:

$$[\log F_{t,t+T} - \log P_t - i_t T] - \delta_0 - \delta_1 \log(INV_t) = 0, \quad (1.9)$$

or using the definition of *IAB*:

$$IAB_t - \delta_0 - \delta_1 \log(INV_t) = 0, \quad (1.10)$$

where the warehousing and insurance cost (*w)* has been absorbed into the intercept.[5]

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[4] Not surprisingly in light of Figures 1 and 2, a quadratic specification $c_t = \beta_0 + \beta_1 INV_t + \beta_2 (INV_t)^2$ also produces regression results with a statistically significant concave relationship between *IAB* and *INV.*

[5] Bresnahan and Suslow (1985), Bresnahan and Spiller (1986), Williams and Wright (1989, 1991), as well as Deaton and Laroque (1991), propose a second version of the theory of storage. It does not rely on the concept of convenience yield, but it generates identical empirical propositions for the relation between the interest-adjusted basis (*IAB*) and the level of inventories. In this version *IAB* is an increasing, concave function of inventories even when the producers and marketers of commodities do not obtain benefits from holding inventories, because the probability of a stock out prior to the expiration of the futures contract varies inversely with inventories. Spot prices exceed forward prices when a stock out happens given the limitation to conduct intertemporal arbitrage transactions. In this context, the spot price has to increase in such a way to equilibrate supply and demand in the spot market. Therefore, when inventories decrease, the probability of a stock out rises, and *IAB* declines. For the purposes of this study, it does not matter which of the two versions of the theory is more likely since the models provide equivalent empirical propositions. As long as there is an increasing, concave relation between *IAB* and inventories, our empirical results are valid regardless of the structural model that generates this reduced form.

II Data Description and Nonparametric Evidence on Hypotheses I and II

Our dataset includes monthly observations from the London Metals Exchange (LME) on spot and three-month futures prices for aluminum, aluminum alloy, copper, lead, nickel, tin, and zinc.[6] We use 90-day future prices (which are quoted in U.S. dollars) and 90-day U.S. T-bill yields throughout. So T =.25 years in all interest-adjusted basis calculations. Prices for copper, aluminum, nickel and zinc cover the period 1988:11 to 2008M07. Prices for lead and tin are available from 1990:1. Prices for the newer aluminum alloy contracts are available from 1993M12. In addition, our dataset includes end-of-month data on LME warehouse inventories for each metal (measured in metric tons). All series were obtained from the Haver Analytics USECON database. Given our absence of data on warehousing and insurance costs, *w* is assumed to be constant over time (or, at least, to vary less than the convenience yield in response to changes in inventory levels).

Figure 2 and Figure 3 show two different graphical perspectives on the relationship between the *IAB* and LME inventory for copper. The time plot in Figure 2 suggests that the variance of the *IAB* is higher when inventory is low in the case of copper. A kernel fitted curve in the scatter plot of these same data in Figure 3 clearly shows evidence in support of Hypothesis I. That is, there is a concave relationship between interest-adjusted basis and inventory. There is also strong visual support for Hypothesis II: the variance around the kernel fitted curve is much larger when inventory is low or *IAB* is negative – alternative indications of a 'tight' market situation. Along the

[6] For an overview of the LME and metal futures contracts, see Crowson (2005). We do not consider cobalt and molybdenum in our analysis because there is not enough price data for these commodities to conduct a proper econometric analysis given that futures contracts for them began to be traded in the LME in February 2010. More details regarding the lead market and its relation with the LME can be reviewed in Keen (2000).

axes of the scatter plot, nonparametric kernel density functions for the variables are also shown.

Figure 2: Interest-Adjusted Basis vs. LME Inventory for Copper 1988M11-2008M07

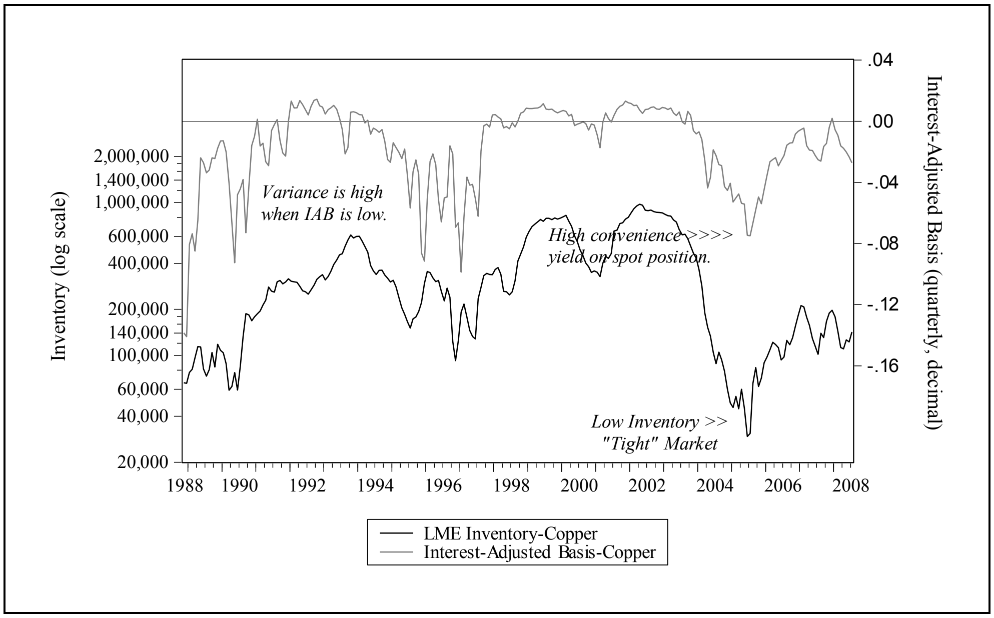


Figure 3: Interest-Adjusted Basis vs. LME Warehouse Inventory for Copper

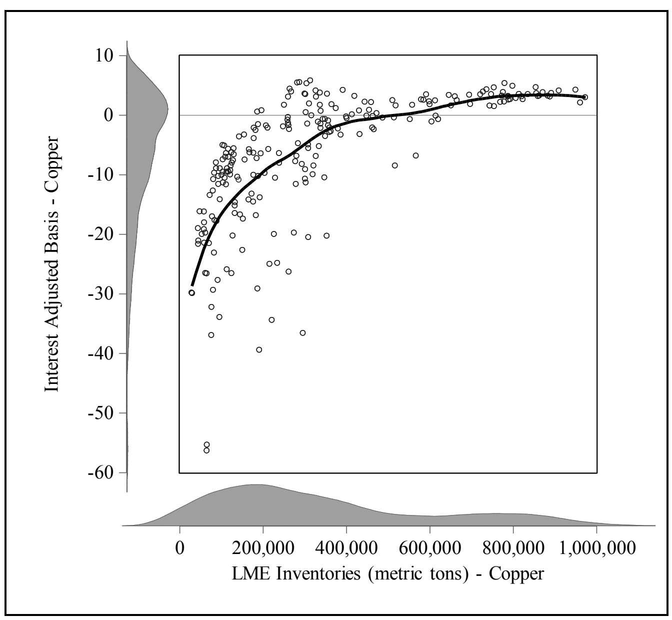


Figure 4 above shows kernel fit scatter plots for the other LME metals: aluminum (AL), aluminum alloy (ALA), lead (PB), nickel (NI), tin (SN) and zinc (ZN). Apart from the aluminum alloy case, which admittedly looks rather odd, the graphs are similar to that for copper. They suggest, for the most part, (1) concave positive relationships between the interest-adjusted basis and inventory, and (2) a variance of the *IAB* that falls as the level of inventory rises (or *IAB* is negative, which is Fama and French's preferred proxy for market tightness).

III Time Series Features of the Data

As a precursor to estimating various single and multiple-equation models to test the four hypotheses discussed in the introduction, it is necessary to (1) carry out unit root tests to assess the order of integration of each series in our dataset and (2) test for cointegration among the series. Table I reports both augmented Dickey-Fuller (ADF) and Phillips-Perron unit root tests on the following series:

$$\begin{aligned} &LF_t \equiv \log(F_{t,t+T}), \\ &LP_t \equiv \log(P_t), \\ &i_t \equiv \log(1+i_{TB,t}), \\ &IAB_t \equiv LF_t - LP_t - .25 * i_t, \\ &BASIS_t = LF_t - LP_t, \\ &LINV_t \equiv \log(INV_t), \end{aligned} \tag{1.11}$$

for each of the seven LME metals. The 90-day T-bill rate is arguably I(0), at least during the low-inflation sample period under consideration. Futures rates and spot rates are clearly I(1), which is consistent with earlier empirical studies.

Figure 4: The Interest Adjusted Basis and Inventory – Other LME Metals

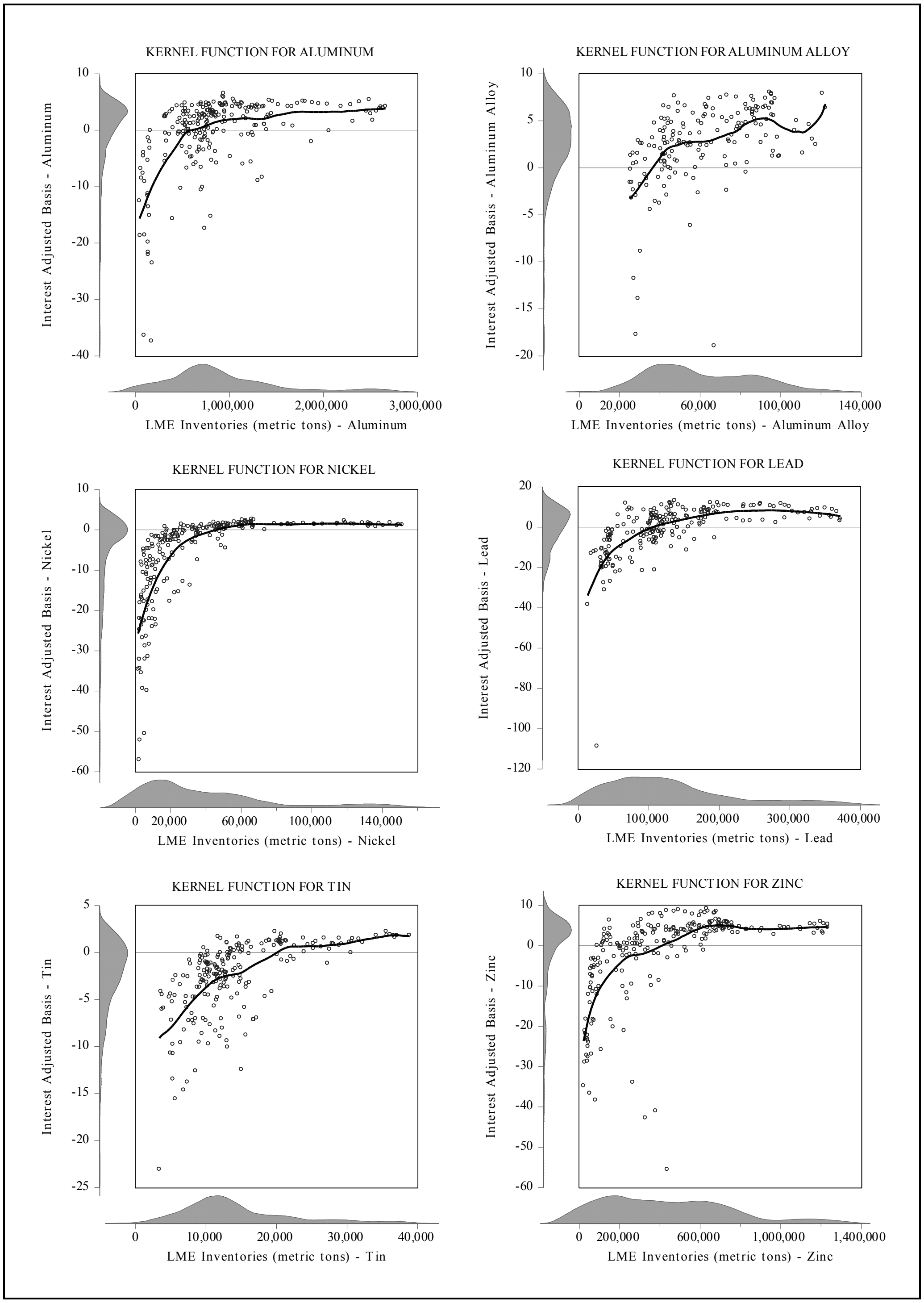


Table I: Unit Root Tests

Sample: 1988M11 (as available) - 2008M07

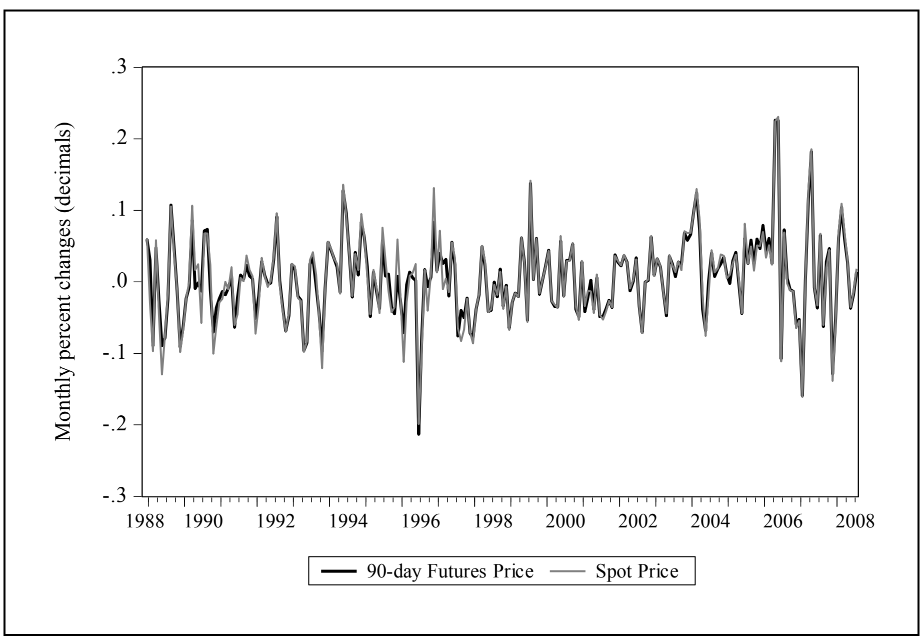
	Augmented Dickey-Fuller Unit Root Tests*				Phillips-Perron Unit Root Tests**			Order of Integration
Series	t-stat.	Prob.	Lags	Obs.	Prob.	Band-width	Obs.	
I	-2.76	0.07	6	230	0.39	9	236	I(0)
LP_AL	-0.98	0.76	2	234	0.73	2	236	I(1)
LP_ALA	-1.16	0.69	1	174	0.64	6	175	I(1)
LP_CU	0.17	0.97	2	234	0.98	1	236	I(1)
LP_NI	-1.39	0.59	2	234	0.69	4	236	I(1)
LP_PB	0.02	0.96	2	220	0.93	3	222	I(1)
LP_SN	1.35	1.00	1	221	1.00	2	222	I(1)
LP_ZN	-1.58	0.49	1	235	0.56	5	236	I(1)
LF_AL	-0.50	0.89	1	235	0.93	4	236	I(1)
LF_ALA	-0.67	0.85	1	174	0.86	5	175	I(1)
LF_CU	0.64	0.99	13	223	1.00	6	236	I(1)
LF_NI	-1.69	0.44	4	232	0.53	2	236	I(1)
LF_PB	-0.02	0.95	13	209	0.78	5	222	I(1)
LF_SN	4.20	1.00	3	219	1.00	8	222	I(1)
LF_ZN	-1.72	0.42	14	222	0.68	0	236	I(1)
IAB_AL	-4.80	0.00	2	234	0.00	10	236	I(0)
IAB_ALA	-4.88	0.00	0	175	0.00	7	175	I(0)
IAB_CU	-2.41	0.14	10	226	0.00	15	236	I(0)/I(1)
IAB_NI	-5.58	0.00	1	235	0.00	4	236	I(0)
IAB_PB	-3.52	0.01	3	219	0.00	3	222	I(0)
IAB_SN	-4.86	0.00	1	221	0.00	4	222	I(0)
IAB_ZN	-3.62	0.01	3	233	0.00	2	236	I(0)
BASIS_AL	-5.21	0.00	2	234	0.00	9	236	I(0)
BASIS_ALA	-4.28	0.00	0	175	0.00	7	175	I(0)
BASIS_CU	-2.37	0.15	10	226	0.00	14	236	I(0)/I(1)
BASIS_NI	-5.84	0.00	1	235	0.00	4	236	I(0)
BASIS_PB	-5.99	0.00	0	222	0.00	3	222	I(0)
BASIS_SN	-4.73	0.00	1	221	0.00	5	222	I(0)
BASIS_ZN	-3.93	0.00	3	233	0.00	0	236	I(0)
LINV_AL	-3.00	0.04	14	222	0.26	7	236	I(0)/I(1)
LINV_ALA	-2.52	0.11	1	186	0.00	8	187	I(0)/I(1)
LINV_CU	-2.62	0.09	12	224	0.22	4	236	I(1)
LINV_NI	-1.99	0.29	2	234	0.10	7	236	I(1)
LINV_PB	-3.06	0.03	14	222	0.35	4	236	I(0)/I(1)
LINV_SN	-2.92	0.04	1	229	0.00	10	230	I(0)
LINV_ZN	-2.83	0.06	14	222	0.09	8	236	I(1)

* Lag Selection Criterion: Akaike Information Criterion.

**Newey-West automatic bandwidth selection and Bartlett kernel.

The null hypothesis of a unit root in the BASIS is clearly rejected for six of the seven metals, with some ambiguity in the case of copper. The same results hold for the interest-adjusted basis (*IAB*). If the short-term interest rate is indeed stationary, these results imply that LF and LP are cointegrated with cointegrating coefficients (1,-1). If the short-term interest rate is nonstationary, the stationarity of *IAB* implies that *LF*, *LP* and *i* are cointegrated with the cointegrating coefficients (1,-1, -0.25). Finally, results for the LME inventory series (*LINV*) series vary, being either I(0) or I(1), depending on the metal and particular unit root tests considered. Financial time series such the log differences of the futures and spot rates (ΔLF, ΔLP) and interest rates are typically characterized by time-varying volatility. Figure 5 for copper is representative of this feature in our dataset.

Figure 5: Monthly Percentage Changes in 90-day Futures and Spot Prices for LME Copper



The validity of the COC model implies that there should be a long-run equilibrium relationship between the four series (*LF*, *LP*, *i*, *LINV*), regardless of whether they are individually I(0) or I(1). We test this hypothesis for each of the seven LME metals by carrying out the Phillips and Ouliaris (1990) cointegration test. This test is similar to the Engle-Granger two-step approach for cointegration in that it performs a unit root test on the OLS residuals from the following regression:

$$LF_t = \delta_1 LP_t + \delta_2 i_t + \delta_3 LINFV_t + \delta_4 + \varepsilon_t . \tag{1.12}$$

Unlike the Engle-Granger test, the Phillips-Ouliaris cointegration test uses a Newey-West robust estimator for the error covariance matrix, which is desirable given the serial correlation and time-varying volatility of our series as well as the residuals in equations like (1.12).[7] Table II reports the Phillips-Ouliaris cointegration results, which uniformly and strongly reject the null hypothesis of no cointegration (the Engle-Granger cointegration produces similar results.) Table III repeats the Phillips-Ouliaris cointegration test considering equation (1.12), but restricts the cointegrating relationship to that implied by the cost-of-carry model where:

$$H_0 : \delta_1 = -1 \ and \ \delta_2 = -0.25 . \tag{1.13}$$

The restricted cointegration test amounts to carrying out the Phillips-Ouliaris test on *IAB=LF-LP-.25*i* and *LINV*. Again the results uniformly and strongly reject the no cointegration null hypothesis in favor of cointegration.[8] With the COC restriction in

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[7] In this sense, the Phillips-Ouliaris test is robust to heteroskedasticity and serial correlation. Another good feature of the test is that in large samples it has superior power properties than the Engle-Granger ADF test. In addition to the Phillips-Oularius cointegration tests reported here, we carried out Johansen cointegration tests. These test results also point strongly towards the presence of cointegration. See Appendix for details.

[8] Note that if the nominal interest rate and the log of inventories are I(0) series, then the finding in Table 1 that the basis is stationary is sufficient to produce a stationary relationship among the four series (*LF*,*LP*, *i*, *LINV*). If *LINV* is I(1), on the other hand, the finding of cointegration among the four series is consistent with the COC model, but at odds with the basis and *IAB* being stationary, ignoring the influence of *LINV*.

(1.13), these test results are consistent with the cost-of-carry model for the seven LME metals (Hypothesis I).

Table II: Phillips-Ouliaris Cointegration Tests for the LME Metals
H_0: No Cointegration Involving (*LF*, *LP*, *i*, *LINV*)

Metal	Tau Stat.	p-value
Copper	-6.623	0.000
Aluminum	-7.182	0.000
Alum. Alloy	-5.727	0.000
Lead	-8.823	0.000
Nickel	-7.579	0.000
Tin	-5.642	0.000
Zinc	-8.085	0.000

Table III: Phillips-Ouliaris Cointegration Test for the LME Metals
H_0: No Cointegration Involving (*IAB*, *LINV*)
This null hypothesis imposes the COC restrictions in (1.13)

Metal	Tau Stat.	p-value
Copper	-6.464	0.000
Aluminum	-7.229	0.000
Alum. Alloy	-5.720	0.000
Lead	-8.933	0.000
Nickel	-6.798	0.000
Tin	-5.672	0.000
Zinc	-7.540	0.000

IV Estimating the COC Model with a GARCH Error Process

Unlike stock prices, spot and futures rates for metals, as well as their basis and *IAB*, often exhibit significant first-order serial correlation, even after log-differencing to remove unit roots. Therefore we allow for flexible adjustment dynamics toward the COC equilibrium condition in (1.10) by estimating an autoregressive distributed lag or ADL(1,2) model:[9]

$$IAB_t = \beta_1 + \beta_2 IAB_{t-1} + \beta_3 \log LINV_{t-1} + \beta_4 \log LINV_{t-2} + \varepsilon_t . \qquad (1.14)$$

It is straightforward to show that this ADL(1,2) specification can be rewritten in an equivalent error-correction form: $\Delta IAB_t = \gamma_1[IAB_{t-1} - \delta_0 - \delta_1 \log(INV_{t-1})] + \gamma_1 \Delta \log LINV_{t-1} + \varepsilon_t$ To capture the time-varying volatility in the error process in (1.14), we estimate the ADL model with a GARCH error process[10] that includes the Fama-French market tightness dummy variable D_t:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda_1 \sigma_{t-1}^2 + \delta D_t + u_t . \qquad (1.15)$$

The innovations in the error process are assumed to have a Student-t distribution, thereby allowing for the 'fat tails' that are a feature of many financial time series. GARCH specifications with three alternative measures of market tightness are also considered. These include a Fama-French-like dummy on the raw (rather than interest-adjusted) basis, and either the series *LINV* or *IAB* (rather than a dummy based on its sign):

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[9] Higher-order terms for *IAB* and *LINV* were not statistically significant and the correlograms for the residuals showed no evidence of remaining serial correlation.

[10] Evans and Guthrie (2008) show that the observation of serial correlation in commodity prices and GARCH characteristics in the variance of these prices is consistent with a competitive storage model of commodity prices featuring frictions (i.e., a cost is incurred each time a unit of the commodity is moved into or out of storage) which introduce an element of irreversibility into storage decisions. In their model the convenience yield is interpreted as the amount by which the expected return from holding a timing real option embedded in each unit of the stored commodity exceeds the opportunity cost of maintaining stocks.

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \lambda_1\sigma_{t-1}^2 + \delta\, LINV_t + u_t$$
$$or \qquad (1.16)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \lambda_1\sigma_{t-1}^2 + \delta\, IAB_t + u_t.$$

All four GARCH specifications produce empirical results that confirm Hypothesis 1 and strongly support Hypothesis 2. Table IV summarizes the ADL(1,2)-GARCH(1,1) results for seven LME metals by reporting the GARCH specifications that minimize the Akaike information criterion.[11]

Table IV: ADL(1,2)-GARCH(1,1) Models for Testing Hypotheses I and II

Dependent Variable:	IAB_CU (Copper)		IAB_AL (Aluminum)		IAB_ALA (Aluminum Alloy)		IAB_NI (Nickel)	
Sample:	1989M01 - 2008M07		1989M01 - 2008M07		1994M01 - 2008M07		1989M01 - 2008M07	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Mean Equation:								
C	-0.030	0.001 ***	-0.011	0.080 *	-0.015	0.082 *	0.000	0.249
IAB(-1)	0.826	0.000 ***	0.577	0.000 ***	0.676	0.000 ***	0.868	0.000 ***
LINV(-1)	0.018	0.000 ***	0.018	0.000 ***	0.018	0.000 ***	0.004	0.000 ***
LINV(-2)	-0.016	0.000 ***	-0.017	0.000 ***	-0.016	0.000 ***	-0.004	0.000 ***
Variance Equation:								
C	0.000	0.257	0.000	0.000 ***	0.000	0.000 ***	0.000	0.018 **
RESID(-1)^2	0.406	0.000 ***	0.216	0.001 ***	0.242	0.014 **	0.516	0.000 ***
GARCH(-1)	0.551	0.000 ***	--	--	--	--	0.216	0.000 ***
Dummy(IAB<0)	0.000	0.020 **	0.000	0.000 ***	0.000	0.021 **	--	--
Dummy(BASIS<0)	--	--	--	--			0.000	0.000 ***
Regression Fit								
R-squared	0.812		0.581		0.594		0.790	
Adjusted R-squared	0.810		0.575		0.586		0.787	
Akaike info criterion	-7.070		-7.790		-8.103		-7.878	
Schwarz criterion	-6.937		-7.672		-7.958		-7.745	
Wald Tests								
Wald1: β3=β4=0	47.612	0.000 ***	29.534	0.000 ***	31.224	0.000 ***	60.690	0.000 ***
Wald2: β3+β4=0	10.316	0.001 ***	5.189	0.023 **	4.436	0.035 **	0.110	0.741

*** Significance at 99% level, ** significance at 95% level, * significance at 90% level. Continues…

[11] Both criteria always choose the same model when the number of estimated parameters is the same in all specifications, as is the case here.)

Table IV: ADL(1,2)-GARCH(1,1) Models for Testing Hypotheses I and II

Dependent Variable:	IAB_PB (Lead)		IAB_SN (Tin)		IAB_ZN (Zinc)	
Sample:	1990M02 - 2008M07		1990M02 - 2008M07		1989M01 - 2008M07	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
Mean Equation:						
C	-0.020	0.096 *	-0.010	0.010 ***	-0.010	0.061 *
IAB(-1)	0.821	0.000 ***	0.808	0.000 ***	0.739	0.000 ***
LINV(-1)	0.026	0.000 ***	0.007	0.000 ***	0.023	0.000 ***
LINV(-2)	-0.024	0.000 ***	-0.006	0.000 ***	-0.022	0.000 ***
Variance Equation:						
C	0.000	0.000 ***	0.000	0.098 *	0.000	0.001 ***
RESID(-1)^2	0.330	0.017 **	0.406	0.015 **	0.480	0.014 **
GARCH(-1)	--	--	0.230	0.035 **	--	--
Dummy(IAB<0)	--	--	0.000	0.000 ***	--	--
Dummy(BASIS<0)	--	--	--	--	0.001	0.007 ***
Regression Fit						
R-squared	0.565		0.743		0.627	
Adjusted R-squared	0.559		0.740		0.622	
Akaike info criterion	-6.483		-8.533		-7.370	
Schwarz criterion	-6.361		-8.395		-7.252	
Wald Tests						
Wald1: $\beta3=\beta4=0$	44.454	0.000 ***	129.031	0.000 ***	30.885	0.000 ***
Wald2: $\beta3+\beta4=0$	3.328	0.068 *	6.923	0.009 ***	5.096	0.024 **

H1: The Interest-Adjusted Basis (IAB) is a positive concave function of the level of inventories. H2: When the market is tight, the IAB is more variable.

*** Significance at 99%, ** significance at 95%, * significance at 90%.
Source: Authors' calculations.

The estimated model for copper in the first column of Table IV provides strong support for the cost-of-carry model (Hypothesis I). See the estimated coefficients for the 'mean equation,' which show that the interest-adjusted basis is a concave function of the level of inventory (in both the short run and the long run). The coefficients on the first and second lags of *LINV* are individually highly significant. We also carried out two Wald tests (Wald1 and Wald2, respectively) to test the following null hypotheses on the coefficients on *LINV* and $LINV_{t-1}$ in (1.14):

$$
\begin{aligned}
&H_0 : \beta_3 = \beta_4 = 0 \\
&and \\
&H_0 : \beta_3 + \beta_4 = 0.
\end{aligned}
\tag{1.17}
$$

The Wald test statistics are reported at the bottom of Table IV. Both indicate strong rejections, as would be expected from the COC model.

Next, consider the GARCH specification in the estimated ADL-GARCH model for copper, which is shown under 'Variance Equation' in Table IV. The best specification based on the model selection criteria used the Fama-French dummy as the indicator of market tightness. The coefficient on the dummy is positive and highly significant, providing strong support for Hypothesis II (that the conditional variance of *IAB* is higher when the market is tight).

The remaining columns in Table IV show that the findings for all seven metals are qualitatively similar to those for copper (and this holds regardless of which measure of market tightness is used in the GARCH specification). The only minor difference is for nickel where Wald1 is strongly rejected, but Wald 2 cannot be rejected (p=0.741).

In summary, given (i) the graphical evidence in the scatter plots in Figures 3 and 4, (ii) the econometric evidence based on the Phillips-Ouliaris cointegration tests and (iii) the estimated ADL(1,2)-GARCH models in Table IV we conclude that the two hypotheses involving the interest-adjusted basis (Hypotheses 1 and 2) are strongly supported by the data for the seven LME metals. That is, (1) there is a strong positive, concave relationship between *IAB* and inventory, as the COC model predicts, and (2) the conditional variance of *IAB* is much higher when the market is 'tight.' The latter results are robust to the alternative proxies for market tightness.

V A VEC-MGARCH Model of Spot and Futures Prices

Up to this point, we have only examined the interest-adjusted basis, not the separate movements of (logs of) futures and spot prices of LME metals, *LF* and *LP*. Following Ng and Pirrong (1994)[12] and Benavides (2010), this section turns to the joint time series behavior of *LF* and *LP*, including their time-varying covariance structure.

Ng and Pirrong use a two-step approach to estimate a bivariate error correction model with an M-GARCH error process for spot and forward prices. They consider four industrial metals traded on the LME (aluminum, copper, lead, and zinc) plus silver, whose characteristics as a precious metal are perhaps more important than its industrial demand. They use daily data from September 1,1986 to September 15, 1992 (except for the aluminum data which began I august 27, 1987). They use weekly warehousing fees in 38 LME warehouses in 12 countries to get a proxy for warehousing costs. On the other hand, they do not use LME inventory data as a proxy for market tightness, relying instead on the adjusted spread. In contrast, we use LME inventory data as a key source of information in our estimation. [13] Also, we estimate the mean and variance-covariance equations simultaneously rather than using the two-step approach in Ng and Pirrong. Like Ng and Pirrong, our methodology properly handles the issues of the non-stationary behavior of time series price data that may generate spurious results during the statistical testing of the FFS hypotheses, as well as the problem of time-varying volatilities also

---

[13] Ng and Pirrong (1994) begin by summarizing two variants of the theory of storage (or cost-of-carry model) that are observationally equivalent for the hypotheses that they test. The first variant relates the interest-and-storage-adjusted basis, which they call the 'adjusted spread,' to the convenience yield. The second is assumed to be an increasing concave function of the level of inventory. The second variant does not rely on the construct of a convenience yield. Instead it assumes that the probability of an inventory stock out increases as the level of inventories falls, causing the market risk premium associated with stock outs to rise. They use the adjusted spread as a proxy.

observed in commodity prices that may affect the efficiency of our hypothesis-testing exercise.

Our ultimate objective is to test generalizations of Hypothesis III (the Samuelson Hypothesis), namely the ***conditional variance*** of future prices is less than the ***conditional variance*** of spot prices, and Hypothesis IV (the Fama-French Hypothesis) which states that when the market is tight, the ***conditional correlation*** is lower than when market is not tight. In the later case, the conditional correlation is near unity.

Our approach is to estimate the following bivariate vector error correction model (VEC) for (*LF*, *LP*), which embodies the long-run equilibrium relationship among the four series in (1.12) and the time-varying volatility of the series *LF* and *LP*:

$$\begin{aligned}\Delta LF_t &= \alpha_{0f} + \alpha_{1f}[LF_{t,t+T} - LP_t - 0.25i_t - \delta_3 - \delta_4 LINV_t] \\ &\quad + \sum_{k=1}^{K} \beta_k^{ff} \Delta LF_{t-k} + \sum_{k=1}^{K} \beta_k^{fs} \Delta LP_{t-k} + \sum_{k=1}^{K} \beta_k^{fi} \Delta i_t + \sum_{k=1}^{K} \beta_k^{fI} \Delta LINV_t + e_t^f \\ \Delta LP_t &= \alpha_{0s} + \alpha_{1s}[LF_{t,t+T} - LP_t - 0.25i_t - \delta_3 - \delta_4 LINV_t] \\ &\quad + \sum_{k=1}^{K} \beta_k^{sf} \Delta LF_{t-k} + \sum_{k=1}^{K} \beta_k^{ss} \Delta LP_{t-k} + \sum_{k=1}^{K} \beta_k^{si} \Delta i_t + \sum_{k=1}^{K} \beta_k^{sI} \Delta LINV_t + e_t^s .\end{aligned} \tag{1.18}$$

The error covariance matrix exhibits time-varying volatility:

$$Var\begin{bmatrix} e_t^f \\ e_t^p \end{bmatrix} \equiv H_t \equiv \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}, \tag{1.19}$$

It is assumed to follow an MGARCH error process with the diagonal VECH(1,1) form proposed in Bollerslev, Engle, and Wooldridge (1988). We add the Fama-French market tightness dummy D_t to the bivariate MGARCH specification to capture the effects of market tightness on the conditional variances of *LF* and *LP*, as well as their covariance:

$$\begin{aligned} h_{11t} &= M_{11} + A_{11}\varepsilon_{1t-1}^2 + B_{11}h_{11t-1} + E_{11}D_t \\ h_{12t} &= A_{12}\varepsilon_{1t-1}\varepsilon_{2t-1} + B_{12}h_{12t-1} + E_{12}D_t \\ h_{22t} &= M_{22} + A_{22}\varepsilon_{2t-1}^2 + B_{22}h_{22t-1} + E_{22}D_t. \end{aligned} \tag{1.20}$$

M_{ij}, A_{ij}, B_{ij}, and E_{ij} are the MGARCH parameters to be estimated. In order to guarantee the positive definiteness of the variance-covariance matrix H_t, we will employ a diagonal parameterization for constant matrix M and a rank one[14] parameterization for the matrices A, B and E – i.e. the ARCH, GARCH, and dummy terms in (1.20)– following Ding and Engle (2001). The VEC-MGARCH model with VECH(1,1) structure is estimated using a maximum likelihood procedure that assumes the errors terms have a multivariate Student-t distribution.[15, 16] Table V shows the estimation results for the VEC(1)-MGARCH(1,1)[17] model for the LME metals. Focus first on the estimated MGARCH processes. Note that the MGARCH specification captures well the time-varying volatility in the ΔLF and ΔLP series, as well as the time-varying covariance effect.[18]

[14] The rank one parameterization is consistent with a financial single-factor model. It can be shown that the variances and covariance of two assets X_1 and X_2 are the following: $cov(X_1, X_2) = \beta_1\beta_2\sigma^2_M$, $var(X_1) = \beta_1^2\sigma^2_M$ and $var(X_2) = \beta_2^2\sigma^2_M$ where β_i is the beta coefficient of asset "*i*" and σ^2_M is the variance of the market index. In matrix notation we get:

$$\text{var}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} [\beta_1 \ \beta_2]\sigma_M^2 = \begin{bmatrix} \beta_1^2 & \beta_1\beta_2 \\ \beta_1\beta_2 & \beta_2^2 \end{bmatrix}\sigma_M^2.$$

[15] See Laurent, Bauwens and Rombouts (2006) for further details regarding the MGARCH specification.

[16] To identify the various constant terms in the VEC, we impose the restriction that *LF* and *LP* have the same stochastic trend: $\alpha_{0s} = \alpha_{0f}$.

[17] The Schwarz criterion applied to the unrestricted VAR with *LF* and *LP* chose two lags. Rewriting the VAR in the canonical or VECM form reduces the lag length by one.

[18] We do not analyze the correlation between different commodity markets and market indices such as the LME Base Metals Index. Interested readers on the subject can review Watkins and McLeer (2005).

Table V: Results of the Estimation of the VEC-MGARCH Model

	COPPER		NICKEL		LEAD		ZINC	
	Mean Equation		Mean Equation		Mean Equation		Mean Equation	
Long-run COC relation	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
LP_{t-1}	-1.000	constr.	-1.000	const	-1.000	constr.	-1.000	constr.
i_{t-1}	-0.250	constr.	-0.250	const	-0.250	constr.	-0.250	constr.
$LINV_{t-1}$ (δ_1)	-0.015 ***	-9.478	0.020	0.464	-0.020 ***	-7.366	-0.005 **	-2.444
Constant in CE (δ_0)	0.195 ***	9.495	-0.198	-0.439	0.220 ***	6.688	0.052 **	2.008
Short-run Forward Dynamics	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
Speed of Adjustment (α_{1f})	0.229	1.517	0.204	0.823	-0.499 ***	-3.268	0.492 ***	4.767
ΔLF_{t-1}	0.659 **	2.529	0.733	1.626	0.202	0.965	-0.011	-0.073
ΔLP_{t-1}	-0.299	-1.241	-0.435	-0.975	-0.064	-0.320	0.196	1.495
Δi_{t-1}	1.907 *	1.713	-0.053	-0.028	-0.158	-0.124	2.668 ***	2.825
ΔINV_{t-1}	-0.046 **	-2.176	-0.007	-0.341	-0.118 ***	-4.664	-0.090 ***	-3.901
Short-run Spot Dynamics	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
Speed of Adjustment (α_{1s})	0.484 ***	3.149	0.225	0.822	-0.298 *	-1.741	0.712 ***	6.455
ΔLF_{t-1}	0.581 **	2.150	0.777	1.625	0.298	1.255	0.023	0.136
ΔLP_{t-1}	-0.217	-0.861	-0.481	-1.010	-0.143	-0.623	0.167	1.126
Δi_{t-1}	2.060 *	1.792	-0.762	-0.386	0.030	0.022	2.653 ***	2.756
ΔINV_{t-1}	-0.060 ***	-2.901	-0.014	-0.718	-0.140 ***	-5.087	-0.109 ***	-4.486
Constant ($\alpha_{0f} = \alpha_{0s}$)	0.005 **	1.992	0.013 **	2.174	-0.004	-1.413	0.004 **	2.555
	MGARCH Equations		MGARCH Equations		MGARCH Equations		MGARCH Equations	
MGARCH Coefficients	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
M(1,1)	0.000001	0.110	0.000112 ***	2.744	-0.000005	-0.830	0.000004	1.009
M(2,2)	0.000000	-0.049	-0.000116 ***	-2.711	0.000002	0.243	-0.000005	-0.940
A1(1,1)	0.256314 ***	3.947	0.430672 **	2.427	0.247796 **	2.037	0.535763 ***	4.681
A1(1,2)	0.253614 ***	3.964	0.453559 **	2.440	0.270907 **	2.053	0.561002 ***	4.696
A1(2,2)	0.250942 ***	3.964	0.477661 **	2.450	0.296174 **	2.069	0.587430 ***	4.693
B1(1,1)	0.767500 ***	20.058	0.628903 ***	19.501	0.822093 ***	93.539	0.623155 ***	19.426
B1(1,2)	0.771741 ***	20.785	0.641422 ***	21.411	0.811179 ***	130.695	0.615183 ***	19.239
B1(2,2)	0.776005 ***	21.383	0.654191 ***	23.468	0.800411 ***	160.032	0.607312 ***	18.797
E1(1,1)	0.000257 **	2.219	0.002233 **	2.198	0.000743 *	1.729	0.000152	1.127
E1(1,2)	0.000234 **	2.119	0.002269 **	2.200	0.000751 *	1.691	0.000095	0.847
E1(2,2)	0.000213 **	2.019	0.002305 **	2.198	0.000760 *	1.647	0.000059	0.674
Adj. R2 Forward Equation	0.103		0.083		0.100		0.112	
Adj. R2 Spot Equation	0.146		0.088		0.111		0.174	
Degrees of Freedom (t-Student)	4.535		4.734		2.833		5.219	
Sample	1989M01 - 2008M07		1989M01 - 2008M07		1990M03 - 2008M07		1989M01 2008M07	
Observations	235		235		221		235	

*** significance at 99% level, ** significance at 95%, * significance at 90%, const: constrained parameter

Continues…

Table V: Results of the Estimation of the VEC-MGARCH Model

	ALUMINUM		TIN		ALUMINUM ALLOY	
	Mean Equation		Mean Equation		Mean Equation	
Long-run COC relation	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
LP_{t-1}	-1.000	constr.	-1.000	constr.	-1.000	constr.
i_{t-1}	-0.250	constr.	-0.250	constr.	-0.250	constr.
$LINV_{t-1}$ (δ_1)	-0.005 ***	-3.933	-0.009 ***	-5.924	-0.004	-1.249
Constant in CE (δ_0)	0.066 ***	3.551	0.093 ***	5.974	0.039	0.989
Short-run Forward Dynamics	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
Speed of Adjustment (α_{1f})	0.558 ***	2.703	-0.517	-1.334	0.185	0.965
ΔLF_{t-1}	0.414	1.300	0.970 **	2.165	0.744 ***	2.946
ΔLP_{t-1}	-0.199	-0.650	-0.776 *	-1.778	-0.485 **	-1.970
Δi_{t-1}	1.476	1.609	-0.226	-0.203	0.314	0.420
ΔINV_{t-1}	-0.031	-1.433	-0.057 ***	-3.130	-0.088 ***	-5.380
Short-run Spot Dynamics	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
Speed of Adjustment (α_{1s})	0.937 ***	3.891	-0.293	-0.724	0.417 *	1.842
ΔLF_{t-1}	0.318	0.859	0.955 **	1.990	0.737 ***	2.638
ΔLP_{t-1}	-0.107	-0.295	-0.765	-1.630	-0.471 *	-1.704
Δi_{t-1}	1.546	1.607	-0.281	-0.242	0.180	0.227
ΔINV_{t-1}	-0.049 **	-2.029	-0.065 ***	-3.528	-0.100 ***	-5.769
Constant ($\alpha_{0f} = \alpha_{0s}$)	-0.001	-0.503	-0.003	-1.207	0.002	0.975
	MGARCH Equations		MGARCH Equations		MGARCH Equations	
MGARCH Coefficients	Estimates	z-stat.	Estimates	z-stat.	Estimates	z-stat.
M(1,1)	-0.000007	-1.252	0.000001	0.452	0.000002	0.419
M(2,2)	0.000007	1.234	-0.000001	-0.318	-0.000002	-0.297
A1(1,1)	0.437048 ***	2.892	0.418563 **	2.298	0.428379 ***	3.372
A1(1,2)	0.447288 ***	2.912	0.422175 **	2.310	0.424827 ***	3.395
A1(2,2)	0.457767 ***	2.927	0.425818 **	2.321	0.421304 ***	3.397
B1(1,1)	0.661199 ***	15.873	0.710734 ***	14.665	0.668204 ***	14.430
B1(1,2)	0.644917 ***	15.378	0.706370 ***	14.692	0.670255 ***	15.105
B1(2,2)	0.629036 ***	14.788	0.702033 ***	14.680	0.672313 ***	15.405
E1(1,1)	0.000889 **	2.239	0.000315 *	1.870	0.000033	0.652
E1(1,2)	0.001086 **	2.314	0.000346 *	1.892	0.000068	0.864
E1(2,2)	0.001327 **	2.389	0.000381 *	1.913	0.000141	1.250
Adj. R2 Forward Equation	0.088		0.080		0.154	
Adj. R2 Spot Equation	0.136		0.086		0.166	
Degrees of Freedom (t-Student)	3.768		3.070		4.689	
Sample	1989M01 - 2008M07		1990M03 2008M07		1994M02 - 2008M07	
Observations	235		221		174	

*** significance at 99%, ** significance at 95%, * significance at 90%
Source: Authors' estimations.

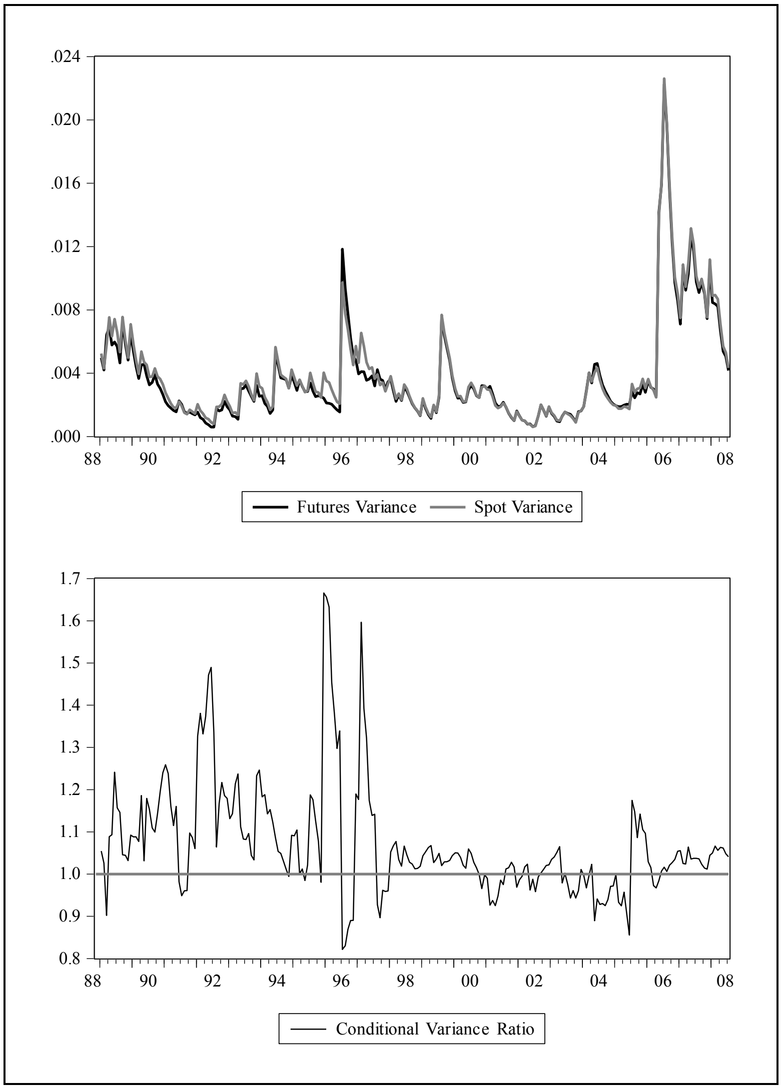
Moreover, the MGARCH coefficients on the Fama-French market tightness dummy E(1,1) and E(2,2) are significant in the conditional variance equations for ΔLF

and ΔLP, as well as their conditional covariance equation (see $E(1,2)$) in the cases of copper, aluminum, nickel, lead and tin. We find no evidence regarding the effect of the market tightness indicator in the cases of zinc and aluminum alloy in our multivariate analysis. See the estimated E_{ij} coefficients in Table V.

We can now test our generalization of the Samuelson hypothesis (Hypothesis III) that the *conditional* variance of the spot price of LME copper exceeds the *conditional* variance of the futures price. Figure 6 shows time plots of the estimated conditional variances for the spot and future copper prices as an example. The lower panel shows the ratio of the conditional variance of the spot rate to the conditional variance of the futures prices. The Samuelson hypothesis holds when this ratio exceeds one.

We observe that the conditional variance ratio is greater than one for 74% of the sample. Is 0.74 statistically different from 0.50? As a formal test of Hypothesis III, we construct a bivariate variable D_SAM_t that equals one when conditional variance ratio > 1 and zero otherwise. The expected value of this dummy $E[D\_SAM_t]$ is the proportion of the time that the Samuelson hypothesis holds. We then test the null hypothesis H_0: $E[D\_SAM_t] \leq 0.50$, which indicates a 50-50 chance of validating/rejecting Samuelson hypothesis. Under the null, there is no statistical support for Samuelson hypothesis. The alternative hypothesis $E[D\_SAM_t] > 0.50$ implies evidence favoring Samuelson hypothesis. The higher is $E[D\_SAM_t] - 0.50$, the stronger is the support for the Samuelson hypothesis. We construct a one-tailed t-statistic based on the difference between the actual proportion, *S*%, and the 50% proportion to carry out the hypothesis test. The results of this test for the seven LME metals are shown in Table VI.

Figure 6: Conditional Spot and Future Variances in the Case of Copper



We observe that the proportion of the sample in which the conditional variance ratio is greater than one is over 75% in the case of copper and aluminum alloy, whereas it is greater than 85% in the cases of aluminum, lead, nickel, tin and nickel. The t-statistic is highly significant in all cases. These results strongly validate the generalized Samuelson

Hypothesis.[19] The conditional variances of spot prices are higher than the conditional variances of the future prices well over 75% of the time in the case of all LME metals.

Table VI: Tests of Samuelson Hypothesis (III)
(H_0: $S\%$ = 0.5 vs. H_1: $S\%$ > 0.5)

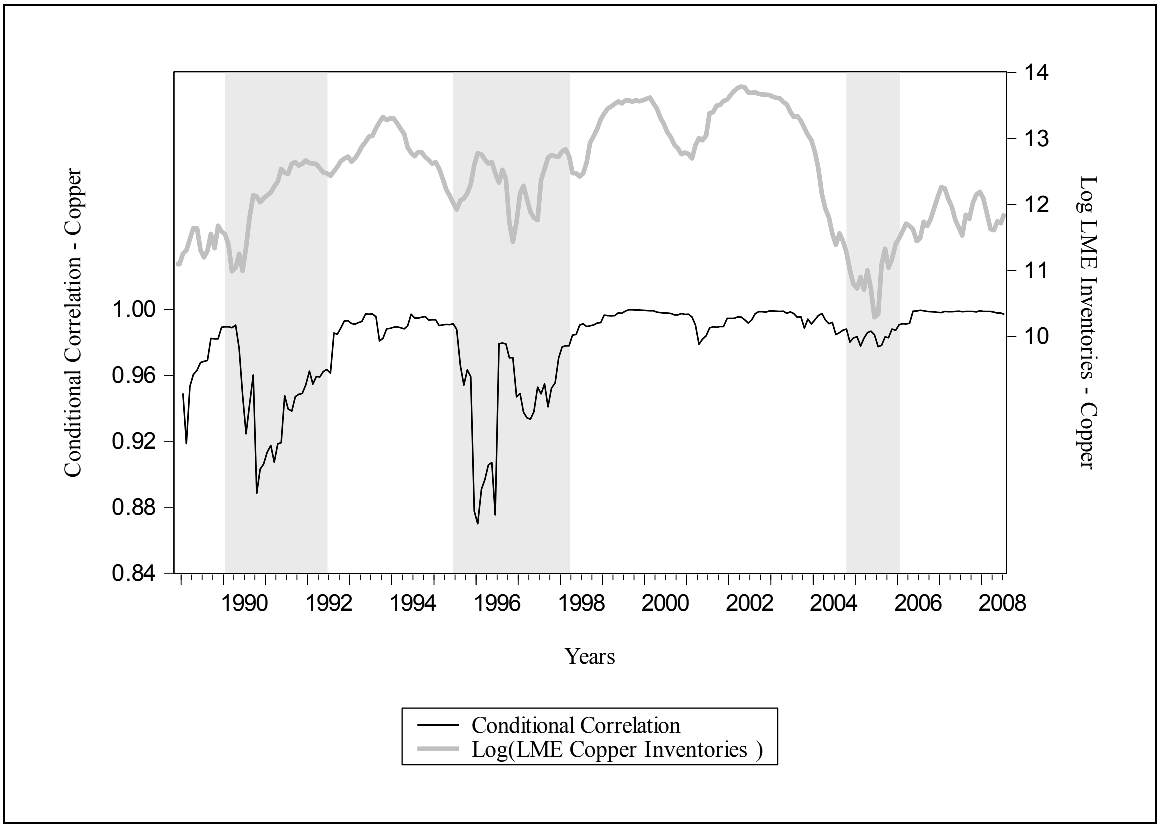
Metal	S% of sample in which the hypothesis holds	Obs.	z-stat.	One-tail p-value
Copper	74.0%	235	8.39	0.00
Aluminum	86.0%	235	15.83	0.00
Aluminum Alloy	76.4%	174	8.19	0.00
Lead	95.0%	221	30.71	0.00
Nickel	88.9%	235	17.98	0.00
Tin	87.8%	221	17.11	0.00
Zinc	86.4%	235	16.23	0.00

Regarding Fama-French Hypothesis IV, Figure 7 shows some graphical evidence. It is possible to observe that in the case of copper when the market was tight in the last decades (meaning a decrease in LME copper inventories), the conditional correlation between the futures and spot prices decreased. Figure 7 identifies three periods of market tightness (shaded areas). The first episode at the beginning of the 1990s coincides with the Persian Gulf War in Kuwait. The 1995-8 period included the Mexican debt crisis of 1994-1995, the Asian crisis of 1997, and the Russian banking crisis of 1998. Finally, the

[19] Our results contrast with those found by Evans and Guthrie (2008) using simulation experiments. Evans and Guthrie reject Samuelson's hypothesis using univariate GARCH (1,1) models for simulated spot and futures prices. The validation of the Samuelson's hypothesis in our case supports the idea that market frictions are not affecting much the dynamics of LME metal prices. Evans and Guthrie would suggest otherwise.

third episode occurred between 2004 and 2006, which Radetzki (2006) views as the start of the most recent commodity boom.

Figure 7: Conditional Correlation of Spot and Futures Prices vs. Log(Inventories): LME Copper



To test the generalized Fama-French Hypothesis IV, we proceeded as follows: (1) re-estimate the VEC-MGARCH model in (1.20) *without* including the market tightness dummy; (2) calculate the conditional correlation, denoted $\sigma_{FP}(t)$, using the estimated conditional variances and covariance; (3) estimate an appropriately constrained Tobit regression relating conditional correlation $\sigma_{FP}(t)$ to the market tightness dummy:

$$\sigma_{FP}(t) = \alpha + \beta D_t + u_t . \qquad (1.21)$$

where the left-hand side variable is constrained (censored) to the interval [-1, +1]. Using (1.21), we test the null hypothesis that the market tightness dummy has no negative impact on the conditional correlation between ΔLF and ΔLP: $H_0 : \beta \geq 0$. Hypothesis IV predicts that $\beta < 0$ in (1.21) and that $\alpha \approx 1$, indicating that the correlation is almost perfect when $D_t = 0$. Table VII shows the results of testing the null hypotheses from the Tobit regression for the seven LME metals:

Table VII: Tests of Fama-French Hypothesis (IV)
Tobit Regressions

Metal	α	t-stat. H_0: $\alpha = 1$		β	t-stat. H_0: $\beta \geq 0$	
Copper	0.985	-6.358	***	-0.020	-4.741	***
Aluminum	0.985	-6.855	***	-0.013	-3.269	***
Aluminum Alloy	0.969	-4.688	***	0.006	0.669	
Lead	0.974	-10.074	***	-0.010	-1.410	
Nickel	0.999	-5.472	***	-0.014	-8.045	***
Tin	0.996	-6.736	***	-0.004	-5.593	***
Zinc	0.981	-4.843	***	-0.022	-2.848	***

We observe that the estimated conditional correlations when the market is not tight (i.e., α) are very high, but given their very small standard errors are statistically different from one. The average for the seven LME metals is 0.984. This suggests that the spot and future LME prices are 'almost perfectly correlated' when the market is not tight, as Fama and French predict. The Fama-French dummy (i.e., β) has the negative sign that Fama and French predict in six cases, and it is statistically significant at 99% level for copper, aluminum, nickel, tin and zinc. The dummy is insignificant in the cases of aluminum alloy and lead. The magnitude of the market tightness effect on conditional correlation, however, is numerically small. On average, it equals -0.014 (excluding aluminum alloy, which has a positive coefficient). As Figure 7 suggests, the conditional correlations are typically very high whether or not the market is tight. In this sense, the empirical relevance of the generalized Fama-French hypothesis is perhaps modest.

VI Conclusions

This paper has presented interesting tests of the four Fama-French-Samuelson hypotheses outlined in the Introduction. We have found strong nonparametric and parametric empirical evidence to support the cost-of-carry model (Hypothesis I), which predicts a concave long-run equilibrium relation between the interest-adjusted basis and inventory. This suggests that inventory responses distribute the effects of demand and supply shocks between spot and futures LME metal prices. There is also very compelling evidence to suggest that the volatility of the *IAB* is higher when the market is tight (Hypothesis II). We provide an empirical specification that allows us to test both of these hypotheses, namely an ADL model with a GARCH error process.

Percentage changes in futures prices and spot prices (as opposed to the basis or percentage gap between them) are much more difficult to explain. We estimate a vector-error correction model for logs of futures and spot rates with a multivariate GARCH error process. This model allows us to test a generalized version of the Samuelson hypothesis, namely the conditional variance of the spot rate is higher that the conditional variance of the futures price. The empirical evidence for seven LME-traded metals is strongly supportive of this hypothesis. Turning to the Fama and French refinement of the Samuelson hypothesis, however, our results are more mixed. We find that the conditional correlation between percentage changes in futures and spot prices is very high (well over 0.9) regardless of whether markets are tight or not. There was only a small reduction in this correlation when markets were tight, with the correlation typically remaining in exceed of 0.9. Thus, for the LME metals, at least, the Fama-French hypothesis does not seem to be particularly important.

The testing methods developed here can be applied in a straightforward way to other primary commodities. Future research will determine the extent to which our empirical findings carry over to them as well.

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Appendix: Additional Cointegration Tests

Section IV of the text reports Phillips-Ouliaris cointegration tests. We also considered the Johansen cointegration test (trace statistic) for each metal. This is a VAR system based test that assumes homogeneous error processes. The null hypothesis is that there are zero cointegrating relationships among the four variables. In all cases, our VECMs on which the Johansen tests are based allow for stochastic trends in each series and one lagged difference of each variable (as suggested by Schwarz criterion). These results, shown in Table A-I, uniformly reject the no cointegration null hypothesis in favor of the cointegration alternative, as the COC model predicts. The Johansen maximum eigenvalue test produced identical conclusions.

Table A-I: Johansen Cointegration Tests for LME Metals

Metal	Trace Statistics	5% Critical Value	p-value **
Copper	76.558	47.856	0.000
Aluminum	86.913	47.856	0.000
Aluminum Alloy	59.605	47.856	0.003
Lead	95.046	47.856	0.000
Nickel	86.882	47.856	0.000
Tin	68.680	47.856	0.000
Zinc	108.756	47.856	0.000

**MacKinnon, Haug and Michelis' (1999) p-values